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A Review of GNSS Carrier Phase Ambiguity Resolution and Conceptual AI-Driven Approaches †

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Abstract: Precise Point Positioning (PPP) with Integer Ambiguity Resolution (IAR) is an effective way of improving the overall performance of the PPP technique both in terms of convergence time and accuracy. Even though PPP-IAR has seen a tremendous development in the last two decades, there is still room for improvement. Specifically, the search stage of candidate carrier phase ambiguities is characterized as an NP-hard problem that requires a long processing time, leading to limitation in the reliability of the identified optimal solution. A field that would have an impact on the search process of carrier phase ambiguities, and is addressed conceptually in this paper, is Artificial Intelligence (AI).

Keywords: Precise Point Positioning; Integer Ambiguity Resolution; Artificial Intelligence; GNSS

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1. Introduction

Precise Point Positioning (PPP) is a GNSS technique employed for obtaining a high-accuracy positioning solution using minimal infrastructure and data corrections [1]. Nevertheless, a major drawback of the method is the long convergence time that for the single constellation case is of an order of 30 min [2]. There are many ways to reduce this time, albeit upon circumstances most of them are deemed cost-inefficient [2]. Among them, a prominent approach resides on Integer Ambiguity Resolution (IAR), which is proven to reduce the convergence time and improve the accuracy of the estimated solution [3]. If the ambiguities are resolved correctly to their integer counterparts, then the carrier phase measurements can be used as precise pseudoranges.

A variety of applications including road ITS (Intelligent Transportation Systems) could benefit from PPP-IAR even if they do not ask for cm level accuracy. Specifically, applications for which the PPP-IAR is exercised, in general can attain robustness and reliability in the estimated solution, leading to a high integrity performance [3]. Moreover, in [4], the impact of PPP-IAR was investigated in urban- and deep-urban environments where satellite obstruction occurs more frequently. The analysis determined that employing dual frequency PPP-IAR can yield better solutions than PPP float solutions for which more satellites and/or constellations are enabled. This suggests that processing an ever-increasing number of satellites might not be the answer in order to substantially improve the PPP performance.

Considering the importance of PPP-IAR, a great number of variations in the technique have been proposed and tested since early 2000. Representations of a PPP-IAR range from the Single-Constellation, Dual-Frequency models of Linear Combinations to the recently developed Multi-Constellation, Multi-Frequency ones that rely on "Undifferenced" and Uncombined GNSS measurements. These developments have decreased the convergence time of PPP-IAR but still, instantaneous convergence while assuring correct IAR is difficult to achieve [5].

A field that might have an impact on fast and reliable PPP with IAR is Artificial Intelligence (AI). Notwithstanding the fact that the mathematical models describing the

GNSS measurements are well defined, AI, and particularly its subset Machine Learning (ML), has been an area of increased research interest for GNSS [6]. According to published work in GNSS research, the most typical applications that increasingly make use of ML include satellite signal acquisition exploiting Multi-Layer Perception (MLP), the Convolutional Neural Network (CNN), NLOS (Non-Line of Sight)/multipath/evil waveform detection and classification via Logistic Regression (LR), Support Vector Machines (SVMs), CNN, Recurrent Neural Networks (RNN), etc., included Earth observation/monitoring harnessing the Artificial Neural Network (ANN); GNSS position error estimation utilizing the Long Short-Term Memory (LSTM) network [7]; the detection of falsely resolved integer ambiguities using ANN or CNN [8]; the prediction of ionospheric corrections; the detection of ionospheric scintillation, the improvement of the satellite clock, orbit prediction accuracy, and parameter prediction in missing GNSS corrections [9]; the calibration of the PVT algorithm's filter parameters (e.g., tuning of the Extended Kalman Filter (EKF)) [10]; and the enhancement of the accuracy of outputted PVT information from the positioning module [11] by deploying Reinforcement Learning (RL).

However, it is realized that current ML approaches have not been used for the IAR problem. Even though it is not explicitly reported in the literature, possible reasons for that could be that most ML techniques require a large amount of data and excessive computational resources for training the algorithms. Moreover, the models developed relying on ML might not perform sufficiently in real-time scenarios due to the lack of representative data, considering that most training sets comprise simulated data. However, instead, another AI approach that belongs to the family of computational intelligence [12] has been tested for tackling the IAR problem. Specifically, computational intelligence algorithms, or metaheuristics [12,13], have been used successfully, but not exhaustively, for resolving the ambiguities to their integer counterparts in the ambiguity domain for relative precise positioning GNSS.

Consequently, accounting for the importance of IAR for a fast-converging, high-integrity solution in challenging GNSS environments, this paper offers a brief state-of-the-art review on PPP-IAR. Next, the role of Artificial Intelligence (AI) in GNSS is explored, focusing on metaheuristic algorithms. Finally, after a summary on the limitations of PPP-IAR is provided, the possible integration schemes of AI-IAR are discussed, including their pros and cons and the generic methodology that shall be adopted in future implementations.

2. Precise Point Positioning with Integer Ambiguity Resolution

In PPP, by exploiting dual-frequency GNSS measurements, it is feasible to solve the ambiguities to their integer counterparts, subject to additional state space representation (SSR) corrections, precise orbits, and clocks made available to the user via a network of (global or regional) permanent stations [1]. Currently, six PPP-IAR (or else known as PPP-RTK) models are available, namely the Common Clocks model 1 (CC-1), the Distinct Clocks model (DC), the Common Clocks model 2 (CC-2), the Integer Recovery Clock model (IRC), the Decoupled Clock Model (DCM), and the Uncalibrated Phase Delays (UPD)/Fractional Cycle Biases (FCB) model [1].

The main differences among the aforementioned models lie (i) in the choice of the S-basis, (ii) in the choice of reparameterization, and (iii) in the choice of handling the ionospheric delay [1]. CC-1, DC, and CC-2 employ an Uncombined (UC) representation of the measurements, while IRC, DCM, and UPD/FCB adopt an Ionosphere-Free (IF) representation of the observations. From the literature, it is well known that adopting linear combinations for the representation of the measurements can (i) eliminate the ionospheric delay (first order) and (ii) amplify the corresponding wavelengths, meaning that the ambiguities could be resolved easier. Analytically, for the IRC, DCM, and UPD/FCB techniques, the procedure for IAR in dual-frequency PPP consists of two steps resolving first the Hatch–Melbourne–Wübbena Wide-Lane (WL) ambiguities, which then enables the resolving of the Narrow-Lane (NL) ambiguities. Step one is mandatory, considering that the IF ambiguities exhibit a relatively small wavelength (of an order of 0.6 cm), whereas when decomposed to WL and NL (which practically is equivalent to an L1 ambiguity [14]), the corresponding wavelengths are 86 cm and 11 cm long, respectively.

In the sequel, as the evolution of GNSS at space, network, and user side became more pervasive, a more elegant deterministic model started to become more frequently adopted, which relied on UC GNSS measurements. In the UC case, the ionospheric delay is handled as an unknown parameter, which enables the utilization of ionospheric constraints. If the ionospheric constraints exhibit better accuracy than the pseudoranges, then a better performance is expected in terms of accuracy and convergence than the LC models; otherwise, they are treated as equivalent [14]. For example, the authors in [15–17] adjusted the UPD/FCB model for UC representation of the GNSS measurements to perform IAR. Furthermore, the authors in [17] stress out the ability of the UC model to achieve rapid re-convergences by exploiting the estimated ionospheric delays.

In addition to leveraging precise ionospheric corrections, another approach for reducing the convergence time of PPP is to use multi-frequency signals. By introducing one additional step to the two-step procedure of dual-frequency PPP-IAR for triple-frequency PPP-IAR, the convergence time can be reduced significantly. Analytically, in this case, first the Extra-Wide-Lane (EWL) ambiguities are resolved to their integer counterparts, followed by the settlement of WL and afterwards the NL ambiguities. This additional step reduces the convergence time of triple-frequency PPP-IAR as a consequence of the long wavelength of the EWL ambiguities (e.g., 5.86 m, 9.77 m, and 4.88 m for GPS, Galileo, and BeiDou, respectively [18]), despite their excessive noise. For instance, the authors in [2] formulated the UPD/FCB model for UC observations, without ionospheric corrections, for triple-frequency, multi-constellation PPP with IAR for instantaneous positioning. It was shown that the concentricity of the N1 residuals within 1 cycle is 90.2% due to the extremely fast ambiguity resolution of the EWL, and subsequently, the instantaneous convergence of the WL ambiguities with a confidence level of 95%, for horizontal and vertical thresholds of 5 cm and 10 cm, respectively.

In addition, triple-frequency, additional frequencies (four and even five frequencies) can be exploited for PPP-IAR as they can provide more choices for EWL and WL combinations [19]. However, since earlier research demonstrated that triple-frequency and quadruple-frequency PPP-IAR perform similarly in accuracy and convergence time [19], some authors aimed to enhance the performance of dual- and triple-frequency PPP-IAR. Specifically, in [18], a near-optimal linear combination for dual- and triple-frequency PPP-AR was derived utilizing the equivalence of mathematical forms between the UC and LC models, considering that LC provide slightly less computational complexity compared to UC functional models, while a functional model with rank 4 or 3 (i.e., a set of linear independent observables) instead of 6 (uncombined observations) can further reduce the computational burden. The results indicate an average of 10 min convergence time with a success rate of 95% in kinematic mode. Finally, in [20], the authors proposed a PPP-IAR-based model that resides on the extended DCM model of simultaneously processing dual- and triple-frequency UC measurements using all available constellations. Their study concludes that quasi-instantaneous centimeter accuracy-level positioning is possible with a moving receiver.

3. Artificial Intelligence for Integer Ambiguity Resolution

Most of the aforementioned implementations of PPP-IAR algorithms, after revealing the integer nature of the corresponding ambiguities, search for the integer set in the ambiguity domain that would result in a highly accurate solution. Notably, this is due to the remaining unmodeled biases, especially when the raw observables contain high noise [21]. Most of the dual- and/or triple-frequency PPP-IAR implementations mentioned in Section 2 implement the LAMBDA (Least-Squares Ambiguity Decorrelation Adjustment) algorithm [22].

The PPP-IAR approach generally employs the LAMBDA algorithm for resolving the NL ambiguities. This is due to the high correlation degree observed between the

NL ambiguities and their accelerated resolution, which contributes significantly in the reduction in the initialization period [15]. Furthermore, for the case that a full ambiguity resolution (FAR) is not feasible, a subset of the ambiguities is fixed, leading to Partial Ambiguity Resolution (PAR). Nevertheless, despite the profound effectiveness of the LAMBDA method [5], it is still computationally cumbersome because all integer candidates need to be checked. This results in increased search times, particularly as the number of ambiguities and the search space increases [23], whereas the major problem with PAR is the decision on which a set of ambiguities should finally be chosen [24].

The carrier phase ambiguity problem is an NP-hard (Non-Polynomial) problem [25], meaning that no explicit algorithm exists to find the exact solution for the integer ambiguities in polynomial time. The PPP-IAR problem, assuming that "Undifferenced" or Single-Differenced, Uncombined, or Combined deterministic functional models are used, in its linearized form reads as follows [26]:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{a} + \boldsymbol{B}\boldsymbol{b} + \boldsymbol{e} \tag{1}$$

where $\boldsymbol{y} \in R^q$ contains the observed minus computed code and carrier-phase measurements, $\boldsymbol{a} \in Z^n$ represents the integer carrier phase ambiguities with the design matrix $\boldsymbol{A}$, $\boldsymbol{b} \in R^p$ contains the real unknowns with the design matrix $\boldsymbol{B}$, and $\mathbf{e}$ represents the additive noise component. Applying the least-squares criterion to Equation (1) for estimating the unknown parameters yields

$$min(\mathbf{y} - \boldsymbol{A}\boldsymbol{a} - \boldsymbol{B}\boldsymbol{b})^{\boldsymbol{T}} \boldsymbol{Q}_y^{-1}(\boldsymbol{y} - \boldsymbol{A}\boldsymbol{a} - \boldsymbol{B}\boldsymbol{b}) \tag{2}$$

where $\boldsymbol{Q_y}$ is the variance–covariance matrix of the GNSS observations. The minimization problem in Equation (2), referred to as the Mixed Integer Non-Linear Programming (MINLP) problem, is equivalent to solving the following standard Integer Least-Squares problem (ILS) or the Integer Quadratic Problem (IQP) [27]:

$$\check{\boldsymbol{a}} = min(\hat{\boldsymbol{a}} - \boldsymbol{a})^{\boldsymbol{T}} \boldsymbol{Q}_{\hat{\boldsymbol{a}}}^{-1}(\hat{\boldsymbol{a}} - \boldsymbol{a}) \tag{3}$$

where $\hat{\boldsymbol{a}}$ is the float solution of Equation (2) by neglecting the integer property of the ambiguities, $\boldsymbol{Q}_{\hat{\boldsymbol{a}}}$ is the variance–covariance matrix of the estimated float ambiguities, and $\check{\boldsymbol{a}}$ is the integer solution of Equation (3). In the recent past, AI methods originating from the intelligence computation field have been exploited for solving the ILS problem in Equation (3), which in principle is a closest lattice point problem [27]. More specifically, this problem could be solved by employing principles of metaheuristics [13,27]. Metaheuristics are iterative optimization techniques for solving combinatorial problems, i.e., finding an optimal solution among a finite large solution space, such as the IAR problem.

Most of the well-known metaheuristic algorithms that have been implemented to IAR are the Genetic Algorithm (GA), Differential Evolution (DE), Ant Colony Optimization (ACO), and Artificial Fish Swarm Algorithm (AFSA) [27–33]. GA and DE draw their inspiration from the process of natural selection, which adopts the idea of the survival of the fittest [28,33]. Both GA and DE belong to Evolutionary Algorithms (EA) [12]. ACO is based on biological ants' foraging behavior, which uses pheromones as a communication tool [27], while AFSA simulates a variety of ecological behaviors of fish schooling in the water [31,32]. Both ACO and AFSA apply to Swarm Intelligence. GA, DE, ACO, and AFSA all belong to population-based optimization methods, meaning that they operate on a set of solutions (candidates), often called population.

In the context of the IAR problem, GA represents the candidate integers (or decision variables) as chromosomes encoded in particular bit strings [28–30,34], where each chromosome corresponds to an individual and all the individuals together are the population. The most common encodings for candidate integer ambiguities in GA are binary- [34], real- [28], and grey-coding [29]. Real coding does not have the drawbacks of binary-coding, i.e., longer code lengths, larger solution space, and longer time costing [28,30], while graycoding can effectively avoid the hamming cliff (i.e., a situation in which a small alteration in a solution's binary representation can result in a big change in its fitness value) [29]. The DE algorithm represents the decision variables as individuals of integer vectors [33]. In AFSA, the state of each artificial fish is represented by the dimension of the decision variable, i.e., ambiguities [31,32]. In ACO, a candidate solution (artificial ant) is also characterized by the dimension of the decision variable [27].

According to [13], the fundamental idea behind any metaheuristic applied to the IAR problem consists of four stages (Figure 1). Firstly, the ambiguities should be appropriately represented using a model from which an a priori search space $Q(N)$ and initial solution (population) shall be constructed (stage 1). The initial population is mainly generated at random [34] or initiated either via random numbers generated from a normal distribution $X \sim N(\hat{a},\ Q_{\hat{a}})$ [32] or by using a priori information from the baseline vector (in the case of double-difference ambiguities) as a constraint for the generated individuals of candidate integer ambiguities [28]. Depending on how the initial solution and constraints are formed for the optimization problem, it is indicated to use some form of decorrelation of the ambiguities, which enhances the convergence properties of the metaheuristic algorithm to the global minima [28,31–33].

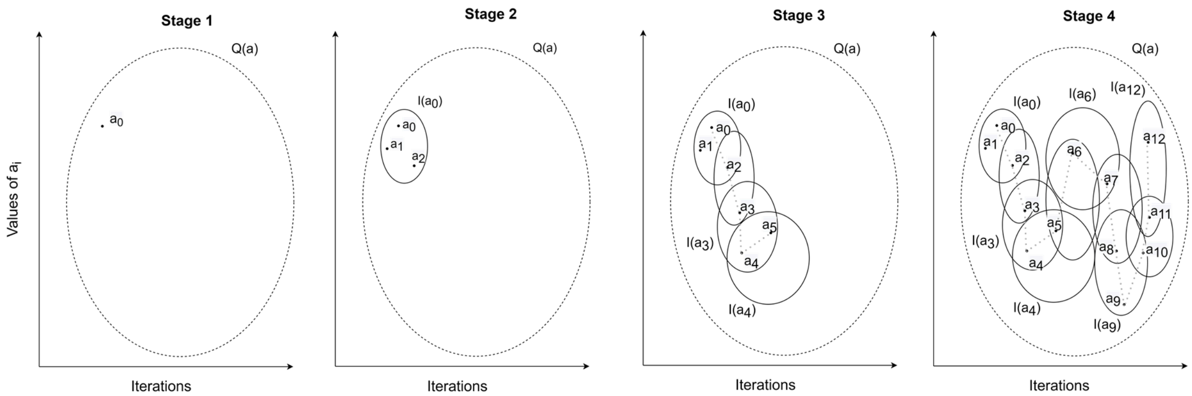

Figure 1. Schematic representation of GNSS ambiguity resolution stages based on metaheuristic principles, modified from [13].

At the following stage (stage 2), a set of neighbor values $I(N_i)$ adjacent to the initial solution is generated by choosing a suitable mechanism. GA and DE utilize crossover, a genetic operation that involves combining genetic information from two parent solutions to produce one or more offspring, and mutation, a genetic operation that introduces small random changes into an individual solution to promote diversity within the population [28–30,33,34]. Despite the fact that both GA and DE use crossover and mutation, in GA, crossover is applied to fixed positions in the encoded string, representing a candidate solution [28–30,34], while on the contrary, DE combines three or more individuals from the population, usually via vector arithmetic [33]. On the other hand, AFSA utilizes the random, preying, swarm, and following behavior of fish [31,32]. Praying corresponds to the behavior of a fish to relocate itself to a location with a higher consecration of food, swarming epitomizes the behavior of fish to avoid danger and over-crowded areas, and following emphasizes the following behaviors of artificial fish when another fish finds a location with a higher concentration of food. Conversely, ACO exploits pheromone levels parametrized using a weighted sum of several one dimensional Gaussian functions, which enables the reconstruction of new solutions [27].

To avoid premature convergence and stagnation at the local minima, some sub-routines of the aforementioned mechanisms undertake explorative behaviors (a series of downhill moves), while others undertake explorative behaviors (a series of uphill moves). In the case of GE and DE, the role of exploitation is adopted via the crossover operation, while exploration is performed via the mutation operation. In AFSA, the random and swarming behaviors of artificial fish give exploration properties to the algorithm, while preying and following act as exploitation mechanisms. ACO implements exploitation by following trails with high-levels of pheromones laid by artificial ants and assures exploration by allowing artificial ants to choose paths with a lower pheromone level as well [27].

Next, the best integer ambiguity candidates are searched and selected based on an acceptable strategy in the vicinity of $I(N_i)$ (stage 3). In GA, DE, and AFSA, searching and selection are applied via the appropriate transformation of the objective function, Equation (2) or Equation (3) [28–33], to a fitness function. In most cases, the transformation involves incorporating a positive scaling factor and applying a logarithmic operation to the original objective function. This transformation occurs in order to avoid premature convergence and trapping in the local optimum. On the contrary, ACO directly uses the objective function of Equation (3) [27]. Finally, the searching process stops when no further improvement is observed (i.e., the population of individuals has converged) [34] or a pre-defined number of iterations is reached (stage 4) [13,27,28,32,33].

The inductions from the several metaheuristic algorithms implemented in the IAR for double-difference ambiguity resolution encompass multiple aspects, including feasibility, a reduction in search space, comparative analysis among different metaheuristic algorithms, and comparison with the LAMBDA algorithm in terms of convergence speed and success rate. Analytically, implementations of GA [28–30,34], DE [33], ACO [27], and AFSA [31,32] demonstrate the feasibility of achieving convergence to the global optimum for the IAR problem. As a case in point, in [27], ACO was used, exploiting a sufficiently large dataset of simulated double-differenced float ambiguities with high precision (inferred via the Ambiguity Delusion of Precision, or ADoP [27]). In these conditions, ACO was able to converge to the global optima for a variety of ambiguity dimensionalities, e.g., 3–20, 21–30, and 31–40, with increasing convergence time [27].

Regarding the reduction in search space, an early implementation of the binary-coded GA implemented in [34] achieved the global optimum for one unknown ambiguity in just over 2 s of CPU processing while exploring something less than 0.7 of the total search space. In [28], a real-coded GA was introduced with adaptive probabilities for crossover and mutation based on individual fitness (Adaptive Genetic Algorithm, AGA). With the same experimental conditions (population number, maximum iterations, etc.), real-coded AGA outperformed the binary-coded GA in terms of iterations needed for convergence, and subsequently, the total search space required. Furthermore, in [31], the total space required for searching for the correct integer ambiguities with AFSA accounts for only 2.18% of the total problem space (41^3).

The authors in [32] used simulated double-difference ambiguities to compare AFSA and AGA. Ensuring that the experimental conditions were the same for both algorithms, they found that AFSA converged faster (6.37 s) compared to AGA (12.92 s). Later, the authors in [31] improved AFSA by adding an attenuation factor to the visual field and an adaptive step size for the artificial fish for better global and local exploration. A simple comparison with GA showed that AFSA can converge to the optimum solution in 2.5 s, while GA requires 5.8 s. Lastly, a recent implementation of an adaptive DE (ADE), where mutation and crossover gained a more adaptive behavior leading to robustness and reliability, outperformed other metaheuristic algorithms like DE and GA [33].

As for comparing metaheuristic algorithms with LAMBDA [5], a few attempts have been realized. Specifically, in [28], the real-coded AGA was compared to LAMBDA in terms of required convergence time. Utilizing a snapshot of low-dimensional simulated double-differenced ambiguities, it was shown that LAMBDA converges under 91.5 s and real-coded AGA under 14.7 s. An evaluation this time of the success rate of LAMBDA and ADE was implemented in [33], using a more representative simulated dataset of 14-dimensional float ambiguities of 100 epochs in succession. The success rate achieved by ADE was 94%, while for LAMBDA, it was 81%. For both AGA [28] and ADE [30] algorithms, the fitness function was subjected to the baseline constraint.

4. Discussion/Proposed Methodology

Undoubtedly, the development of PPP models and algorithms for IAR has seen tremendous progress in the last two decades. Today, it is possible to achieve a highly accurate and robust position solution within a few minutes or quasi-instantaneously when multi-constellation and triple-frequency GNSS measurements are used. Furthermore, a significant improvement in the convergence and robustness of dual-frequency, multi-constellation PPP-IAR is observed. However, there is still room for further improvement in the search process for resolving the correct ambiguities after their integer property is retrieved, in the case of PPP-IAR.

Today, the standard, state-of-the-art methodology for searching integer ambiguities after their integer property is secured is the LAMBDA method [5]. Although it is very effective, it is still computationally complex for the reasons explained in Section 3. In this regard, an interesting alternative for IAR in the ambiguity domain relies on AI, and to this effect, metaheuristics have great potential due to their inherent properties (e.g., parallel search, global optimization, and robustness). Early implementations suggest comparable results to well-established approaches and adaptation, to some extent, to the properties of the specific problem at hand, which is how an optimization algorithm can achieve a better performance than others according to the no-free-lunch theorem [35]. However, all of the existing implementations are designed for double-differenced ambiguities; the results so far are based on simulated instances of them inferred from strong models, and in most cases, the baseline constraint is used to subject the fitness function. Therefore, further investigation is required, utilizing appropriate datasets, undergoing experiments, and exploiting the advancements that have occurred in metaheuristics for solving NP-hard problems (calibration, initialization, noise addition, hybrid metaheuristic algorithms, etc. [12,36,37]).

In this direction, it is envisioned that future research should focus on metaheuristics by studying their impact on searching for integer ambiguities in the ambiguity domain after their integer property is assured. Regarding the latter one, a PPP-IAR method will be selected, considering its efficiency in terms of position quality capabilities (i.e., convergence time and accuracy) and computational resources; specifically, achieving equivalent results by efficiently reducing the dimensionality of the problem (e.g., estimable parameters). The suggested AI approach for PPP-IAR will be assessed via tests involving a set of known elements that are expected to affect its performance (e.g., dimensionality of the unknown ambiguities) and a comprehensive comparison with the LAMBDA algorithm.

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