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Article

Dispersive Optical Solitons to Stochastic Resonant NLSE with Both Spatio-Temporal and Inter-Modal Dispersions Having Multiplicative White Noise

Elsayed M. E. Zayed [1], Mohamed E. M. Alngar [2,*] and Reham M. A. Shohib [3]

1 Mathematics Department, Faculty of Sciences, Zagazig University, Zagazig 44519, Egypt
2 Basic Science Department, Faculty of Computers and Artificial Intelligence, Modern University for Technology & Information, Cairo 11585, Egypt
3 Basic Science Department, Higher Institute of Foreign Trade & Management Sciences, New Cairo Academy, Cario 11835, Egypt
* Correspondence: mohamed.hassan@cs.mti.edu.eg

Abstract: The current article studies optical solitons solutions for the dimensionless form of the stochastic resonant nonlinear Schrödinger equation (NLSE) with both spatio-temporal dispersion (STD) and inter-modal dispersion (IMD) having multiplicative noise in the itô sense. We will discuss seven laws of nonlinearities, namely, the Kerr law, power law, parabolic law, dual-power law, quadratic–cubic law, polynomial law, and triple-power law. The new auxiliary equation method is investigated. We secure the bright, dark, and singular soliton solutions for the model.

Keywords: stochastic; itô calculus; multiplicative noise; solitons

MSC: 34A34; 35C08; 35G20; 35A25; 35G20

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1. Introduction

The stochastic nonlinear differential equations which contain the stochastic term with multiplicative noise play an essential role in scientific fields and engineering. One of these models' fundamental physical problems is getting their soliton solutions. The search for mathematical techniques to deduce exact solutions for these equations is a fundamental action. It is well known that the resonant nonlinear Schrödinger equation (NLSE) comprises the nonlinear dynamics of optical solitons and Madelung fluids. Generally, in the quantum Hall effect, we take into consideration the study of chiral solitons in a specific resonant term in (1 + 1) dimensions [1–8] and in (2 + 1) dimensions [9]. Recently, many papers have deduced the exact solitons solutions for nonlinear partial differential equations (NLPDEs) by using different methods. Namely, Hirota bilinear method [10], physical information neural network (PINN) method [11], Riccati equation expansion method, and Jacobian elliptic equation expansion method [12], semi-inverse variational principle [13], improved adomian decomposition method [14], undetermined coefficients method [15], modified simple equation scheme, and trial equation approach [16], ansatz approach [17], tanh-coth scheme [18], the mapping method based on a Riccati equation [19], and others. Recently, there are new applications of NLSEs such as, the physical information neural network [20], the waveguide amplifier [21], the breather solutions in different planes [22], the comprehended dynamics of solitary waves in the local case [23], and the anti-interference ability of stable solutions [24]. Recently, a number of articles on stochasticity have been published [25–35].

In the article [36], the authors discussed the wick-type stochastic NLSE using the Hirota method combined with the Hermite transformation; howver, in our present article, we have discussed the stochastic resonant NLSE in the itô sense using the new auxiliary equation method. These two governing models are absolutely different.

The current paper focuses on studying the dimensionless form of the stochastic resonant NLSE with both STD and IMD having multiplicative noise in the itô sense with seven different kinds of nonlinear forms. In the recent corresponding Itô calculus, the soliton solutions will be deduced by using the new auxiliary equation method.

Governing Model

The dimensionless form of the stochastic resonant NLSE with both STD and IMD having multiplicative noise in the itô sense is introduced, for the first time, as

$$i\Phi_t + a\Phi_{xx} + b\Phi_{xt} + F\left(|\Phi|^2\right)\Phi + \gamma\left(\frac{|\Phi|_{xx}}{|\Phi|}\right)\Phi + \sigma(\Phi - ib\Phi_x)\frac{dW(t)}{dt} = i\delta\Phi_x, \tag{1}$$

where $\Phi = \Phi(x,t)$ is a complex-valued function symbolizes the wave profile, a, b, γ, δ, and σ are real-valued constants with $i = \sqrt{-1}$. The first term of Equation (1) is the linear temporal evolution, also the chromatic dispersion (CD) and STD terms are symbolized by a and b, respectively. Next, $F\left(|\Phi|^2\right)$ is the functional which represents the nonlinearity forms, while γ is the coefficient of resonant nonlinearity, and δ is coefficient of IMD. Finally, σ is the coefficient of noise strength and $W(t)$ is the standard Wiener tactic such that $\frac{dW(t)}{dt}$ is the white noise. Without noise ($\sigma = 0$), Equation (1) reduces to the well-known resonant NLSE with both STD and IMD which has been previously studied in [7,8]. The motivation for adding the stochastic term $\sigma(\Phi - ib\Phi_x)\frac{dW(t)}{dt}$ to Equation (1) is to formulate the stochastic differential equation with noise or fluctuations depending on the time, which has been recognized in many areas via physics, engineering, biology, chemistry, and so on.

The purpose of the present paper is to derive bright, dark, and singular soliton solutions for Equation (1) with seven various forms of nonlinearity, namely, Kerr law, power law, parabolic law, dual-power law, quadratic-cubic law, polynomial law, and triple-power law by using the new auxiliary equation technique.

In Section 2, we will construct the mathematical analysis for Equation (1). In Sections 3–9, we will establish seven laws of nonlinearities mentioned above for Equation (1) and solving them by using the new auxiliary equation method. In Section 10, conclusions will be presented.

2. Mathematical Analysis

In order to solve the stochastic Equation (1), we use a wave transformation involving the noise coefficient σ and the Wiener process $W(t)$ in the form

$$\Phi(x,t) = g(z)\, e^{i\left[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t\right]}, \tag{2}$$

and

$$z = x - vt, \tag{3}$$

where κ, ω, and v are real constants. Thus, the real function $g(z)$ represents the pulse shape, while κ, ω, and v symbolize to soliton frequency, wave number and soliton velocity, respectively. Inserting (2) and (3) in Equation (1), one deduces

$$(a - bv + \gamma)g'' + \left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]g + F\left(g^2\right)g = 0, \tag{4}$$

and

$$\left[-v - 2a\kappa + b\kappa v + b\left(\omega - \sigma^2\right) - \delta\right]g' = 0. \tag{5}$$

which represent the real and imaginary parts, respectively. From Equation (5), the soliton velocity is obtained as

$$v = \frac{b\left(\omega - \sigma^2\right) - 2a\kappa - \delta}{1 - b\kappa}, \tag{6}$$

provided

$$b\kappa \neq 1. \tag{7}$$

In the next sections, we will solve Equation (4) when $F(\Phi^2)$ takes seven forms of nonlinearities.

3. Kerr Law

To this end, the nonlinearity form of the Kerr law is specified by

$$F\left(g^2\right) = cg^2, \tag{8}$$

such that c is a non-zero constant. Equation (1) using (8) becomes

$$i\Phi_t + a\Phi_{xx} + b\Phi_{xt} + c|\Phi|^2\Phi + \gamma\left(\frac{|\Phi|_{xx}}{|\Phi|}\right)\Phi + \sigma(\Phi - ib\Phi_x)\frac{dW(t)}{dt} = i\delta\Phi_x, \tag{9}$$

Thus, Equation (4) takes the form

$$(a - bv + \gamma)g'' + \left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]g + cg^3 = 0. \tag{10}$$

Now, we will employ the following method to solve Equation (10).

New Auxiliary Equation Approach

To use this method (see [37]), we allow the solution of Equation (10) to be

$$g(z) = \sum_{m=0}^{N} H_m Q^m(z), \tag{11}$$

as long as $Q(z)$ satisfies the ODE

$$Q'^2(z) = \sum_{h=0}^{M} r_h Q^h(z), \quad M \leqslant 8, \tag{12}$$

where H_m and r_h are constants, such that $H_N \neq 0$, $r_M \neq 0$ and N is the balance number which is determined from the formula

$$D\left[g^j g^{(l)}\right] = N(j+1) + l\left(\frac{M}{2} - 1\right).$$

Set $M = 8$, one gets

$$D\left[g^j g^{(l)}\right] = N(j+1) + 3l, \tag{13}$$

which means $D(g) = N$, $D(g^2) = 2N$, $D(g') = N + 3$, $D(g'') = N + 6$ and so on. The current method derives the solutions of Equation (12) as

Family-1 . If $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$, $r_2 > 0$, then one gets

(I) Bright soliton solutions

$$Q(z) = \left(\frac{2\varepsilon r_2}{\sqrt{r_5^2 - 4r_2 r_8}\cosh(3\sqrt{r_2}z) - \varepsilon r_5}\right)^{\frac{1}{3}}, \tag{14}$$

provided $r_5^2 - 4r_2 r_8 > 0$ and $\varepsilon = \pm 1$.

(II) Singular soliton solutions

$$Q(z) = \left(\frac{2\varepsilon r_2}{\sqrt{-\left(r_5^2 - 4r_2 r_8\right)} \sinh(3\sqrt{r_2}z) - \varepsilon r_5} \right)^{\frac{1}{3}}, \tag{15}$$

provided $r_5^2 - 4r_2r_8 < 0$ and $\varepsilon = \pm 1$.

Family-2. If $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$, $r_2 > 0$ and $r_8 = \frac{r_5^2}{4r_2}$, then one gets

(I) Dark soliton solutions

$$Q(z) = \left\{ -\frac{r_2}{r_5} \left[1 \pm \tanh\left(\tfrac{3}{2}\sqrt{r_2}z \right) \right] \right\}^{\frac{1}{3}}. \tag{16}$$

(II) Singular soliton solutions

$$Q(z) = \left\{ -\frac{r_2}{r_5} \left[1 \pm \coth\left(\tfrac{3}{2}\sqrt{r_2}z \right) \right] \right\}^{\frac{1}{3}}. \tag{17}$$

As a result, by using (13), we balance g'' and g^3 in Equation (10), to derive $N = 3$. Consequently, from (11), the solution of Equation (10) has the form

$$g(z) = H_0 + H_1 Q(z) + H_2 Q^2(z) + H_3 Q^3(z), \tag{18}$$

where $H_m (m = 0, 1, 2, 3)$ are constants and $H_3 \neq 0$. Substituting (18) and (12) with $M = 8$ into Equation (10), one derives the following algebraic equations,

$$\left.\begin{aligned}
&18(a - bv + \gamma)H_3 r_8 + cH_3^3 = 0,\\
&20(a - bv + \gamma)H_2 r_8 + 6cH_2H_3^2 + 33(a - bv + \gamma)H_3 r_7 = 0,\\
&(a - bv + \gamma)[4H_2 r_0 + H_1 r_1] + 2cH_0^3 + 2H_0\left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right] = 0,\\
&3cH_2^2H_3 + (a - bv + \gamma)(4H_1 r_8 + 9H_2 r_7 + 15H_3 r_6) + 3cH_1H_3^2 = 0,\\
&3cH_0^2H_1 + (a - bv + \gamma)(6H_3 r_0 + 3H_2 r_1 + H_1 r_2) + \left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]H_1 = 0,\\
&3(a - bv + \gamma)H_1 r_3 + 15(a - bv + \gamma)H_3 r_1 + 6cH_0H_1^2 + 6cH_0^2H_2\\
&+2\left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]H_2 + 8(a - bv + \gamma)H_2 r_2 = 0,\\
&3cH_1^2H_3 + [12H_3 r_4 + 7H_2 r_5 3H_1 r_6](a - bv + \gamma) + 3cH_1H_2^2 + 6cH_0H_2H_3 = 0,\\
&6cH_0H_3^2 + 2cH_2^3 + [7H_1 r_7 + 27H_3 r_5 + 16H_2 r_6](a - bv + \gamma) + 12cH_1H_2H_3 = 0,\\
&12cH_0H_1H_3 + 6cH_1^2H_2 + 6cH_0H_2^2 + [5H_1 r_5 + 12H_2 r_4 + 21H_3 r_3](a - bv + \gamma) = 0,\\
&\left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]H_3 + (5H_2 r_3 + 9H_3 r_2)(a - bv + \gamma) + cH_1^3 + 6cH_0H_1H_2\\
&+3cH_0^2H_3 + 2(a - bv + \gamma)H_1 r_4 = 0.
\end{aligned}\right\} \tag{19}$$

Thus, we utilize the following types of solutions:

Type-1. Set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$, in Equation (19) and solving them by using the Maple, one secures

$$H_0 = 0,\ \ H_1 = 0,\ \ H_2 = 0,\ \ H_3 = 3\sqrt{-\tfrac{2(a-bv+\gamma)r_8}{c}},\ \ r_2 = -\tfrac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{9(a-bv+\gamma)},\ \ r_5 = 0,\ \ r_8 = r_8, \tag{20}$$

provided $c(a - bv + \gamma)r_8 < 0$. Consequently, inserting (20) along with (14) and (15) into Equation (18), one deduces the solutions of Equation (9) as

(I) Bright soliton solutions

$$\Phi(x,t) = \pm\sqrt{-\frac{2\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c}}\,\mathrm{sech}\left[\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{21}$$

provided $\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]c < 0$ and $(a-bv+\gamma)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right] < 0$. (see Figure 1)

(II) Singular soliton solutions

$$\Phi(x,t) = \pm\sqrt{\frac{2\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c}}\,\mathrm{csch}\left[\sqrt{-\frac{\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{a-bv+\gamma}}(x-vt)\right]e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{22}$$

provided $\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]c > 0$ and $(a-bv+\gamma)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right] < 0$. (see Figure 2)

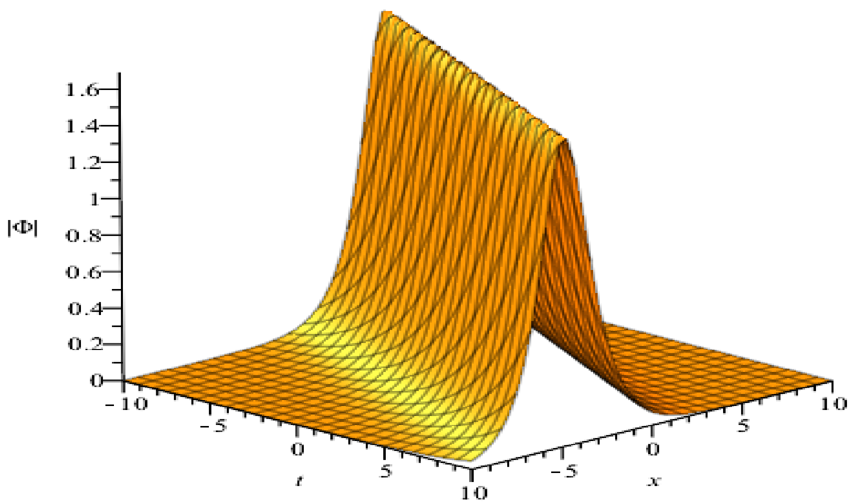

Figure 1. Plot of the bright soliton solution (21) with $a = 0.2$, $b = 0.35$, $c = 0.4$, $\delta = 0.5$, $\kappa = 0.25$, $\omega = 0.6$, $\sigma = 0.35$, $v = -0.4743835616$, $\gamma = 0.5$, and $-10 \le x, t \le 10$.

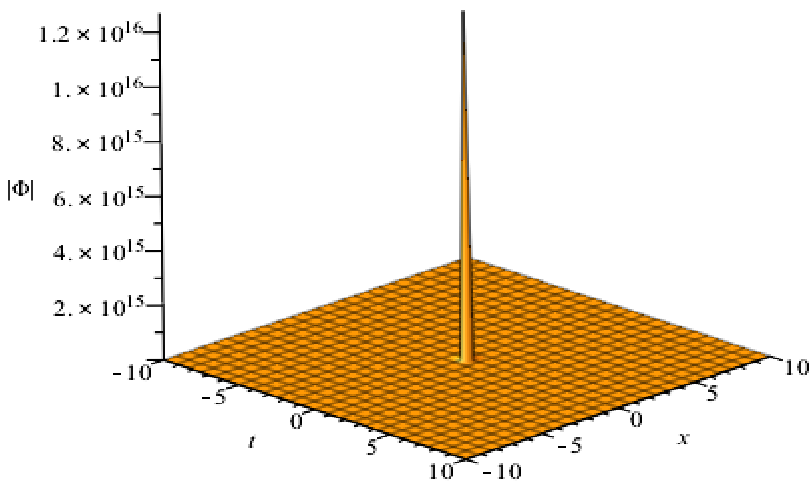

Figure 2. Plot of the singular soliton solution (22) with $a = 0.2$, $b = 0.35$, $c = -0.4$, $\delta = 0.5$, $\kappa = 0.25$, $\omega = 0.6$, $\sigma = 0.35$, $v = -0.4743835616$, $\gamma = 0.5$, and $-10 \le x, t \le 10$.

Type-2. Set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$ and $r_8 = \frac{r_5^2}{4r_2}$, in Equation (19) and solving them by using the Maple, one obtains

$$E_0 = \sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{c}},\ E_1 = 0,\ E_2 = 0,\ E_3 = \frac{9r_5(a-bv+\gamma)}{2[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]}\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{c}},$$
$$r_2 = \frac{2\left[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{9(a-bv+\gamma)},\ r_5 = r_5, \tag{23}$$

provided $c\left[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa\right] < 0$ and $(a-bv+\gamma) \neq 0$. Consequently, inserting (23) along with (16) and (17) into Equation (18), one deduces the solutions of Equation (9) as

(I) Dark soliton solutions (see Figure 3)

$$\Phi(x,t) = \pm\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{c}}\tanh\left[\sqrt{\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{2(a-bv+\gamma)}}(x-vt)\right]e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{24}$$

(II) Singular soliton solutions (see Figure 4)

$$\Phi(x,t) = \pm\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{c}}\coth\left[\sqrt{\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{2(a-bv+\gamma)}}(x-vt)\right]e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{25}$$

provided $c\left[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa\right] < 0$ and $(a-bv+\gamma)\left[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa\right] > 0$.

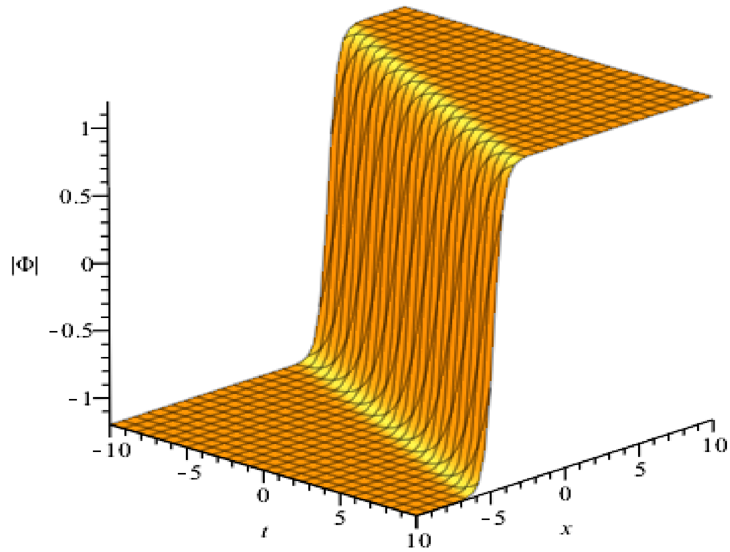

Figure 3. Plot of the dark soliton solution (24) with $a = 0.2$, $b = 0.35$, $c = 0.4$, $\delta = 0.5$, $\kappa = 0.25$, $\omega = 0.6$, $\sigma = 0.35$, $v = -0.4743835616$, $\gamma = -0.5$, and $-10 \leq x, t \leq 10$.

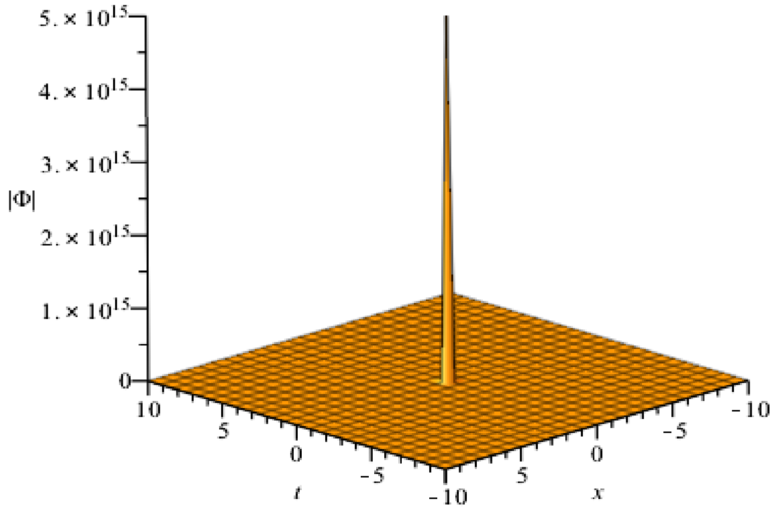

Figure 4. Plot of the singular soliton solution (25) with $a = 0.2$, $b = 0.35$, $c = 0.4$, $\delta = 0.5$, $\kappa = 0.25$, $\omega = 0.6$, $\sigma = 0.35$, $v = -0.4743835616$, $\gamma = -0.5$, and $-10 \leq x, t \leq 10$.

4. Power Law

To this aim, the nonlinearity form of the power law is specified by

$$F\left(g^2\right) = cg^{2n}, \tag{26}$$

such that c is a non-zero constant and n is the power nonlinearity parameter. Equation (1) using (26) becomes

$$i\Phi_t + a\Phi_{xx} + b\Phi_{xt} + c|\Phi|^{2n}\Phi + \gamma\left(\frac{|\Phi|_{xx}}{|\Phi|}\right)\Phi + \sigma(\Phi - ib\Phi_x)\frac{dW(t)}{dt} = i\delta\Phi_x, \tag{27}$$

Thus, Equation (4) takes the form

$$(a - bv + \gamma)g'' + \left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]g + cg^{2n+1} = 0. \tag{28}$$

By using (13), we balancing g'' and g^{2n+1} in Equation (28), to derive $N = \frac{3}{n}$. Since N is not integer, then one takes

$$g(z) = [\varphi(z)]^{\frac{3}{n}}, \tag{29}$$

as long as $\varphi(z) > 0$. Inserting (29) into Equation (28) obtains

$$3(a - bv + \gamma)\left[n\varphi\varphi'' + (3 - n)\varphi'^2\right] + n^2\left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]\varphi^2 + n^2c\varphi^8 = 0. \tag{30}$$

Now, we will employ the following method to solve Equation (30).

New Auxiliary Equation Approach

As a result, by using (13), we balance $\varphi\varphi''$ and φ^8 in Equation (30), deriving $N = 1$. Consequently, from (11), the solution of Equation (30) has the form

$$\varphi(z) = H_0 + H_1 Q(z), \tag{31}$$

where $H_m(m = 0, 1)$ are constants and $H_1 \neq 0$. Substituting (31) and (12) with $M = 8$ into Equation (30), one derives the following algebraic equations,

$$\left.\begin{aligned}
&cn^2H_1^8+9(n+1)(a-bv+\gamma)H_1^2r_8=0,\\
&16cn^2H_0H_1^7+\left[3(5n+3)H_1^2r_7+12H_1nH_0r_8\right](a-bv+\gamma)=0,\\
&112cn^2H_0^5H_1^3+\left[3(n+6)H_1^2r_3+12H_1nH_0r_4\right](a-bv+\gamma)=0,\\
&3(a-bv+\gamma)\left[H_1^2r_4(n+3)+5H_1nH_0r_5\right]+140cn^2H_0^4H_1^4=0,\\
&\left[9H_1nH_0r_3+18H_1^2r_2\right](a-bv+\gamma)+\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]n^2H_1^2+28cn^2H_0^6H_1^2=0,\\
&112cn^2H_0^3H_1^5+\left[9H_1^2nr_5+18H_1nH_0r_6+18H_1^2r_5\right](a-bv+\gamma)=0,\\
&56cn^2H_0^2H_1^6+\left[21H_1nH_0r_7+12H_1^2nr_6+18H_1^2r_6\right](a-bv+\gamma)=0,\\
&2\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa+cH_0^6\right]n^2H_0^2+\left[6H_1^2r_0(3-n)+3H_1nH_0r_1\right](a-bv+\gamma)=0,\\
&3H_1[2nH_0r_2+(6-n)H_1r_1](a-bv+\gamma)+4n^2H_0H_1\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa+4cH_0^6\right]=0.
\end{aligned}\right\} \tag{32}$$

Thus, we utilize the following type of solutions:

Type-1. Set $r_0=r_1=r_3=r_4=r_6=r_7=0$, in Equation (32) and, solving them by using the Maple, obtaining

$$H_0=0,\;\; H_1=\left[-\frac{9(n+1)(a-bv+\gamma)r_8}{n^2c}\right]^{\frac{1}{6}},\;\; r_2=-\frac{n^2\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{9(a-bv+\gamma)},\;\; r_5=0,\;\; r_8=r_8, \tag{33}$$

provided $c(a-bv+\gamma)r_8<0$. Consequently, inserting (33) along with (14) and (15) into Equation (31), one deduces the solutions of Equation (27) as

(I) Bright soliton solutions

$$\Phi(x,t)=\left\{\pm\sqrt{-\frac{(n+1)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c}}\operatorname{sech}\left[n\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{(a-bv+\gamma)}}(x-vt)\right]\right\}^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2t\right]}, \tag{34}$$

provided $\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]c<0$ and $(a-bv+\gamma)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]<0$.

(II) Singular soliton solutions

$$\Phi(x,t)=\left\{\pm\sqrt{\frac{(n+1)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c}}\operatorname{csch}\left[n\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{(a-bv+\gamma)}}(x-vt)\right]\right\}^{\frac{1}{n}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2t\right]}, \tag{35}$$

provided $\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]c>0$ and $(a-bv+\gamma)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]<0$.

5. Parabolic Law

To this aim, the nonlinearity form of the parabolic law is specified by

$$F\left(g^2\right)=c_1g^2+c_2g^4, \tag{36}$$

where c_1 and c_2 are constants and $c_2\neq 0$. Equation (1) using (36) becomes

$$i\Phi_t+a\Phi_{xx}+b\Phi_{xt}+\left(c_1|\Phi|^2+c_2|\Phi|^4\right)\Phi+\gamma\left(\frac{|\Phi|_{xx}}{|\Phi|}\right)\Phi+\sigma(\Phi-ib\Phi_x)\frac{dW(t)}{dt}=i\delta\Phi_x, \tag{37}$$

Thus, Equation (4) takes the form

$$(a-bv+\gamma)g''+\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]g+c_1g^3+c_2g^5=0. \tag{38}$$

By using (13), we balancing g'' and g^5 in Equation (38), to derive $N = \frac{3}{2}$. Since N is not integer, one then takes

$$g(z) = [\varphi(z)]^{\frac{3}{2}}, \tag{39}$$

as long as $\varphi(z) > 0$. Inserting (39) into Equation (38) obtains

$$3(a - bv + \gamma)\left[2\varphi\varphi'' + \varphi'^2\right] + 4\left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]\varphi^2 + 4c_1\varphi^5 + 4c_2\varphi^8 = 0. \tag{40}$$

Next, we employ the following method to solve Equation (40).

New Auxiliary Equation Approach

As a result, by using (13), we balance $\varphi\varphi''$ and φ^8 in Equation (40), to get $N = 1$. Consequently, from (11), the solution of Equation (40) has the same form (31). Substituting (31) and (12) with $M = 8$ into Equation (40), one derives the following algebraic equations,

$$\left.\begin{aligned}
&4c_2H_1^8 + 27(a - bv + \gamma)H_1^2 r_8 = 0,\\
&21(a - bv + \gamma)\left(H_1^2 r_6 + H_1H_0 r_7\right) + 112c_2H_0^2H_1^6 = 0,\\
&32c_2H_0H_1^7 + 24(a - bv + \gamma)\left(H_1H_0 r_8 + H_1^2 r_7\right) = 0,\\
&18(a - bv + \gamma)\left(H_1^2 r_5 + H_1H_0 r_6\right) + 224c_2H_0^3H_1^5 + 4c_1H_1^5 = 0,\\
&20c_1H_0H_1^4 + 280c_2H_0^4H_1^4 + 15(a - bv + \gamma)\left(H_1H_0 r_5 + H_1^2 r_4\right) = 0,\\
&40c_1H_0^2H_1^3 + 224c_2H_0^5H_1^3 + 12(a - bv + \gamma)\left(H_1^2 r_3 + 12H_1H_0 r_4\right) = 0,\\
&6(a - bv + \gamma)\left(H_1^2 r_1 + H_1H_0 r_2\right) + 20c_1H_0^4H_1 + 8\left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]H_0H_1 + 32c_2H_0^7H_1 = 0,\\
&40c_1H_0^3H_1^2 + 4\left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]H_1^2 + 112c_2H_0^6H_1^2 + 9\left(H_1^2 r_2 + H_1H_0 r_3\right)(a - bv + \gamma) = 0,\\
&3(a - bv + \gamma)\left(H_1H_0 r_1 + H_1^2 r_0\right) + 4\left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]H_0^2 + 4c_1H_0^5 + 4c_2H_0^8 = 0
\end{aligned}\right\} \tag{41}$$

Thus, we utilize the following types of solutions:

Type-1. Set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$, in Equation (41) and solving them by using the Maple, one obtains

$$H_0 = 0,\ H_1 = \left[-\frac{27(a-bv+\gamma)r_8}{4c_2}\right]^{\frac{1}{6}},\ r_2 = -\frac{4\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{9(a-bv+\gamma)},\ r_5 = -\frac{c_1}{(a-bv+\gamma)}\sqrt{-\frac{(a-bv+\gamma)r_8}{3c_2}},\ r_8 = r_8, \tag{42}$$

provided $c_2(a - bv + \gamma)r_8 < 0$. Consequently, inserting (42) along with (14) and (15) into Equation (31), one deduces the solutions of Equation (37) as

(I) Bright soliton solutions

$$\Phi(x,t) = \left\{\frac{12\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{\pm\sqrt{9c_1^2-48c_2[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]}\cosh\left[2\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{(a-bv+\gamma)}}(x-vt)\right]-3c_1}\right\}^{\frac{1}{2}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{43}$$

provided $9c_1^2 - 48c_2[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] > 0$ and $(a - bv + \gamma)[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] < 0$.

(II) Singular soliton solutions

$$\Phi(x,t) = \left\{\frac{12\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{\pm\sqrt{-\left(9c_1^2-48c_2[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]\right)}\sinh\left[2\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{(a-bv+\gamma)}}(x-vt)\right]-3c_1}\right\}^{\frac{1}{2}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{44}$$

provided $9c_1^2 - 48c_2[(\omega-\sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa] < 0$ and $(a-bv+\gamma)[(\omega-\sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa] < 0$.

Type-2. Set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$ and $r_8 = \frac{r_5^2}{4r_2}$, in Equation (41) and solving them by using the Maple, one obtains

$$H_0 = 0,\ \ H_1 = \left[\frac{243(a-bv+\gamma)^2 r_5^2}{64c_2[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]}\right]^{\frac{1}{6}},\ \ r_2 = -\frac{4\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{9(a-bv+\gamma)},\ \ r_5 = r_5, \tag{45}$$

and

$$c_1 = -4c_2\sqrt{\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{3c_2}}, \tag{46}$$

provided $c_2\left[\left(\omega-\sigma^2\right)(b\kappa-1) - a\kappa^2 - \delta\kappa\right] > 0$ and $(a-bv+\gamma) \neq 0$. Consequently, inserting (45) along with (16) and (17) into Equation (31), one deduces the solutions of Equation (37) as:

(I) Dark soliton solution

$$\Phi(x,t) = \left\{\frac{1}{2}\sqrt{\frac{3\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c_2}}\left(1+\tanh\left[\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right)\right\}^{\frac{1}{2}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{47}$$

(II) Singular soliton solution

$$\Phi(x,t) = \left\{\frac{1}{2}\sqrt{\frac{3\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c_2}}\left(1+\coth\left[\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right)\right\}^{\frac{1}{2}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{48}$$

provided $c_2[(\omega-\sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa] > 0$ and $(a-bv+\gamma)[(\omega-\sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa] < 0$.

6. Dual Power Law

To this aim, the nonlinearity form of the dual power law is specified by

$$F\left(g^{2n}\right) = c_1 g^{2n} + c_2 g^{4n}, \tag{49}$$

where c_1 and c_2 are constants and $c_2 \neq 0$. Equation (1) using (49) becomes

$$i\Phi_t + a\Phi_{xx} + b\Phi_{xt} + \left(c_1|\Phi|^{2n} + c_2|\Phi|^{4n}\right)\Phi + \gamma\left(\frac{|\Phi|_{xx}}{|\Phi|}\right)\Phi + \sigma(\Phi - ib\Phi_x)\frac{dW(t)}{dt} = i\delta\Phi_x, \tag{50}$$

Thus, Equation (4) takes the form

$$(a-bv+\gamma)g'' + \left[\left(\omega-\sigma^2\right)(b\kappa-1) - a\kappa^2 - \delta\kappa\right]g + c_1 g^{2n+1} + c_2 g^{4n+1} = 0. \tag{51}$$

By using (13), we balancing g'' and g^{4n+1} in Equation (51), to derive $N = \frac{3}{2n}$. Since N is not integer, then one takes

$$g(z) = [\varphi(z)]^{\frac{3}{2n}}, \tag{52}$$

as long as $\varphi(z) > 0$. Inserting (52) into Equation (51) yields

$$3(a-bv+\gamma)\left[2n\varphi\varphi'' + (3-2n)\varphi'^2\right] + 4n^2\left[\left(\omega-\sigma^2\right)(b\kappa-1) - a\kappa^2 - \delta\kappa\right]\varphi^2 + 4n^2 c_1\varphi^5 + 4n^2 c_2\varphi^8 = 0. \tag{53}$$

Next, we will employ the following method to solve Equation (53).

New Auxiliary Equation Approach

As a result, by using (13), we balance $\varphi\varphi''$ and φ^8 in Equation (53), to get $N = 1$. Consequently, from (11), the solution of Equation (53) has the same form (31). Substituting (31) and (12) with $M = 8$ into Equation (53), one derives the following algebraic equations,

$$\left.\begin{array}{l}
9(a-bv+\gamma)(2n+1)H_1^2r_8 + 4c_2n^2H_1^8 = 0,\\[2mm]
3(a-bv+\gamma)\left[H_1^2r_6(4n+3) + 7nH_1H_0r_7\right] + 112c_2n^2H_0^2H_1^6 = 0,\\[2mm]
3(a-bv+\gamma)\left[H_1^2r_7(5n+3) + 8nH_1H_0r_8\right] + 32c_2n^2H_0H_1^7 = 0,\\[2mm]
4\left[\left(\omega-\sigma^2\right)(b\kappa-1) - a\kappa^2 - \delta\kappa\right]n^2H_1^2 + (a-bv+\gamma)\left(9H_1^2r_2 + 9H_1nH_0r_3\right)\\
+8\left(14c_2H_0^3 + 5c_1\right)n^2H_0^3H_1^2 = 0,\\[2mm]
4n^2H_1^5\left(c_1 + 56c_2H_0^3\right) + \left(9H_1^2r_5 + 9H_1^2nr_5 + 18H_1nH_0r_6\right)(a-bv+\gamma) = 0,\\[2mm]
20n^2H_0H_1^4\left(14c_2H_0^3 + c_1\right) + (a-bv+\gamma)\left[3(2n+3)H_1^2r_4 + 15nH_1H_0r_5\right] = 0,\\[2mm]
224c_2n^2H_0^5H_1^3 + 40c_1n^2H_0^2H_1^3 + \left(3H_1^2nr_3 + 12H_1nH_0r_4 + 9H_1^2r_3\right)(a-bv+\gamma) = 0,\\[2mm]
+4c_1n^2H_0^5 + 4c_2n^2H_0^8 + (a-bv+\gamma)\left(3H_1nH_0r_1 - 6H_1^2nr_0 + 9H_1^2r_0\right)\\
+4\left[\left(\omega-\sigma^2\right)(b\kappa-1) - a\kappa^2 - \delta\kappa\right]n^2H_0^2 = 0,\\[2mm]
\left[6H_1nH_0r_2 + 3(3-n)H_1^2r_1\right](a-bv+\gamma) + 20c_1n^2H_0^4H_1 + 32c_2n^2H_0^7H_1\\
+8\left[\left(\omega-\sigma^2\right)(b\kappa-1) - a\kappa^2 - \delta\kappa\right]n^2H_0H_1 = 0.
\end{array}\right\} \quad (54)$$

Thus, we utilize the following types of solutions:

Type-1. Set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$, in Equation (54) and solving it by using the Maple, one obtains

$$\begin{gathered}
H_0 = 0,\ H_1 = \left[-\tfrac{9(2n+1)(a-bv+\gamma)r_8}{4n^2c_2}\right]^{\frac{1}{6}},\ r_2 = -\tfrac{4n^2\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{9(a-bv+\gamma)},\\
r_5 = -\tfrac{2nc_1}{3(1+n)(a-bv+\gamma)}\sqrt{-\tfrac{(2n+1)(a-bv+\gamma)r_8}{c_2}},\ r_8 = r_8,
\end{gathered} \quad (55)$$

provided $c_2(a - bv + \gamma)r_8 < 0$. Consequently, inserting (55) along with (14) and (15) into Equation (31), one deduces the solutions of Equation (50) as

(I) Bright soliton solutions

$$\Phi(x,t) = \left\{\frac{2(1+n)(2n+1)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{\pm\sqrt{(2n+1)^2c_1^2-4(1+n)^2(2n+1)c_2[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]}\cosh\left[2n\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]-(2n+1)c_1}\right\}^{\frac{1}{2n}} \quad (56)$$

$$\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]},$$

provided $(2n + 1)c_1^2 - 4(1 + n)^2c_2[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] > 0$ and $(a - bv + \gamma)[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] < 0$.

(II) Singular soliton solutions

$$\Phi(x,t) = \left\{ \frac{2(1+n)(2n+1)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{\pm\sqrt{-\left\{(2n+1)^2 b_1^2-4(1+n)^2(2n+1)b_2[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]\right\}}\sinh\left[2n\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{(a-bv+\gamma)}}(x-vt)\right]-(2n+1)c_1} \right\}^{\frac{1}{2n}} \tag{57}$$

$$\times e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]},$$

provided $(2n+1)c_1^2 - 4(1+n)^2 c_2[(\omega-\sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa] < 0$ and $(a - bv + \gamma)[(\omega - \sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa] < 0$.

Type-2. Set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$ and $r_8 = \frac{r_5^2}{4r_2}$, in Equation (54) and solving them by using the Maple, one obtains

$$H_0 = 0, \;\; H_1 = \left[\frac{81(2n+1)(a-bv+\gamma)^2 r_5^2}{64n^4[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]c_2}\right]^{\frac{1}{6}}, \;\; r_2 = -\frac{4n^2\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{9(a-bv+\gamma)}, \;\; r_5 = r_5, \tag{58}$$

and

$$c_1 = -2(n+1)c_2\sqrt{\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{(2n+1)c_2}}, \tag{59}$$

provided $c_2\left[\left(\omega-\sigma^2\right)(b\kappa-1) - a\kappa^2 - \delta\kappa\right] > 0$ and $(a - bv + \gamma) \neq 0$. Now, substituting (58) along with (16) and (17) into Equation (31), one derives

(I) Dark soliton solution

$$\Phi(x,t) = \left\{ \frac{1}{2}\sqrt{\frac{(2n+1)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c_2}}\left(1+\tanh\left[n\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right)\right\}^{\frac{1}{2n}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{60}$$

(II) Singular soliton solution

$$\Phi(x,t) = \left\{ \frac{1}{2}\sqrt{\frac{(2n+1)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c_2}}\left(1+\coth\left[n\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right)\right\}^{\frac{1}{2n}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{61}$$

provided $c_2[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] > 0$ and $(a - bv + \gamma)[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] < 0$.

7. Quadratic–Cubic Law

To this end, the nonlinearity form of the quadratic–cubic law is specified by

$$F\left(g^2\right) = c_1 g + c_2 g^2, \tag{62}$$

where c_1 and c_2 are constants. Equation (1) using (62) becomes

$$i\Phi_t + a\Phi_{xx} + b\Phi_{xt} + \left(c_1|\Phi| + c_2|\Phi|^2\right)\Phi + \gamma\left(\frac{|\Phi|_{xx}}{|\Phi|}\right)\Phi + \sigma(\Phi - ib\Phi_x)\frac{dW(t)}{dt} = i\delta\Phi_x, \tag{63}$$

Thus, Equation (4) takes the form

$$(a - bv + \gamma)g'' + \left[\left(\omega - \sigma^2\right)(b\kappa - 1) - a\kappa^2 - \delta\kappa\right]g + c_1 g^2 + c_2 g^3 = 0. \tag{64}$$

Next, we will employ the following method to solve Equation (64).

New Auxiliary Equation Approach

As a result, by using (13), we balance g'' and g^3 in Equation (64), to get $N = 3$. Consequently, from (11), the solution of Equation (64) has the same form (18). Substituting (18) and (12) with $M = 8$ into Equation (64), one derives the following algebraic equations,

$$\left.\begin{array}{l}
18(a-bv+\gamma)H_3r_8+c_2H_3^3=0,\\
(a-bv+\gamma)(20H_2r_8+33H_3r_7)+6c_2H_2H_3^2=0,\\
2H_0\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]+2c_1H_0^2+2c_2H_0^3+(4H_2r_0+H_1r_1)(a-bv+\gamma)=0,\\
3c_2H_1H_3^2+(15H_3r_6+9H_2r_7+4H_1r_8)(a-bv+\gamma)+3c_2H_2^2H_3=0,\\
2c_1H_0H_1+\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]H_1+3c_2H_0^2H_1+(a-bv+\gamma)(6H_3r_0+3H_2r_1+H_1r_2)=0,\\
6c_2H_0H_2H_3+3c_2H_1H_2^2+3c_2H_1^2H_3+2c_1H_2H_3+(12H_3r_4+3H_1r_6+7H_2r_5)(a-bv+\gamma)=0,\\
6c_2H_0H_3^2+2c_2H_2^3+12c_2H_1H_2H_3+(a-bv+\gamma)(16H_2r_6+7H_1r_7+27H_3r_5)+2c_1H_3^2=0,\\
(8H_2r_2+3H_1r_3+15H_3r_1)(a-bv+\gamma)+2c_1H_1^2+2H_2\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]\\
+6c_2H_0^2H_2+4c_1H_0H_2+6c_2H_0H_1^2=0,\\
12c_2H_0H_1H_3+(a-bv+\gamma)(21H_3r_3+12H_2r_4+5H_1r_5)+4c_1H_1H_3+6c_2H_0H_2^2\\
+6c_2H_1^2H_2+2c_1H_2^2=0,\\
2c_1H_1H_2+3c_2H_0^2H_3+c_2H_1^3+6c_2H_0H_1H_2+\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]H_3\\
+2c_1H_0H_3+(2H_1r_4+5H_2r_3+9)(a-bv+\gamma)H_3r_2=0
\end{array}\right\} \quad (65)$$

Thus, we utilize the following types of solutions:

Type-1. Set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$, in Equation (65) and solving them by using the Maple, one obtains

$$H_0=0,\ H_1=0,\ H_2=0,\ H_3=3\sqrt{-\frac{2(a-bv+\gamma)r_8}{c_2}},\ r_2=-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{9(a-bv+\gamma)},\ r_5=-\frac{2c_1}{9}\sqrt{-\frac{2r_8}{(a-bv+\gamma)c_2}},\ r_8=r_8, \quad (66)$$

provided $c_2(a - bv + \gamma)r_8 < 0$. Consequently, inserting (66) along with (14) and (15) into Equation (18), one deduces the solutions of Equation (63) as:

(I) Bright soliton solutions

$$\Phi(x,t)=\left\{\frac{6\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{\pm\sqrt{4c_1^2-18c_2[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]}\cosh\left[\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]-2c_1}\right\}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (67)$$

provided $4c_1^2 - 18c_2[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] > 0$ and $(a - bv + \gamma)[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] < 0$.

(II) Singular soliton solutions

$$\Phi(x,t)=\left\{\frac{6\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{\pm\sqrt{-\left\{4c_1^2-18c_2[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]\right\}}\sinh\left[\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]-2c_1}\right\}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \quad (68)$$

provided $4c_1^2 - 18c_2[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] < 0$ and $(a - bv + \gamma)[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] < 0$.

Type-2. Set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$ and $r_8 = \frac{r_5^2}{4r_2}$, in Equation (65) and solving them by using the Maple, one obtains

$$H_0 = 0,\ H_1 = 0,\ H_2 = 0,\ H_1 = -\tfrac{27(a-bv+\gamma)r_5}{2c_1},\ \ r_2 = -\tfrac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{9(a-bv+\gamma)},\ \ r_5 = r_5, \tag{69}$$

and

$$c_2 = \frac{2c_1^2}{9\left[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}, \tag{70}$$

provided $\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa \neq 0$ and $(a-bv+\gamma) \neq 0$. Consequently, inserting (69) along with (16) and (17) into Equation (18), one deduces the solutions of Equation (63) as

(I) Dark soliton solution

$$\Phi(x,t) = -\tfrac{3\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{2c_1}\left(1+\tanh\left[\frac{1}{2}\sqrt{-\tfrac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right)e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{71}$$

(II) Singular soliton solution

$$\Phi(x,t) = -\tfrac{3\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{2c_1}\left(1+\coth\left[\frac{1}{2}\sqrt{-\tfrac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right)e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{72}$$

provided $c_1 \neq 0$ and $(a-bv+\gamma)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right] < 0$.

8. Polynomial Law

To this end, the nonlinearity form of the polynomial law is specified by

$$F\left(g^2\right) = c_1 g^2 + c_2 g^4 + c_3 g^6, \tag{73}$$

where c_1, c_2 and c_3 are constants and $c_3 \neq 0$. Equation (1) using (73) becomes

$$i\Phi_t + a\Phi_{xx} + b\Phi_{xt} + \left(c_1|\Phi|^2 + c_2|\Phi|^4 + c_3|\Phi|^6\right)\Phi + \gamma\left(\frac{|\Phi|_{xx}}{|\Phi|}\right)\Phi + \sigma(\Phi - ib\Phi_x)\frac{dW(t)}{dt} = i\delta\Phi_x, \tag{74}$$

Thus, Equation (4) takes the form

$$(a-bv+\gamma)g'' + \left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]g + c_1 g^3 + c_2 g^5 + c_3 g^7 = 0. \tag{75}$$

Next, we will employ the following method to solve Equation (75).

New Auxiliary Equation Approach

As a result, by using (13), we balance g'' and g^7 in Equation (75), to get $N = 1$. Consequently, from (11), the solution of Equation (75) has the form

$$g(z) = H_0 + H_1 Q(z), \tag{76}$$

where $H_m(m = 0,1)$ are constants and $H_1 \neq 0$. Substituting (76) and (12) with $M = 8$ into Equation (75), one derives the following algebraic equations,

$$\left.\begin{array}{l}
c_3H_1^7+4(a-bv+\gamma)H_1r_8=0,\\
14c_3H_0H_1^6+7(a-bv+\gamma)H_1r_7=0,\\
10c_2H_0H_1^4+5(a-bv+\gamma)H_1r_5+70c_3H_0^3H_1^4=0,\\
21c_3H_0^2H_1^5+c_2H_1^5+3(a-bv+\gamma)H_1r_6=0,\\
c_1H_1^3+10c_2H_0^2H_1^3+35c_3H_0^4H_1^3+2(a-bv+\gamma)H_1r_4=0,\\
20c_2H_0^3H_1^2+3(a-bv+\gamma)H_1r_3+42c_3H_0^5H_1^2+6c_1H_0H_1^2=0,\\
2c_2H_0^5+2c_3H_0^7+(a-bv+\gamma)H_1r_1+2H_0\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]+2c_1H_0^3=0,\\
5c_2H_0^4H_1+\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]H_1+(a-bv+\gamma)H_1r_2+7c_3H_0^6H_1+3c_1H_0^2H_1=0
\end{array}\right\} \quad (77)$$

Thus, set $r_0=r_1=r_3=r_4=r_6=r_7=0$, in Equation (77) and solving them by using the Maple, one obtains

$$H_0=0,\ H_1=\left[-\frac{4(a-bv+\gamma)r_8}{c_3}\right]^{\frac{1}{6}},\ r_2=-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma},\ r_5=0,\ r_8=r_8, \quad (78)$$

and

$$c_1=0,\ \ c_2=0, \quad (79)$$

provided $c_3(a-bv+\gamma)r_8<0$. Consequently, inserting (78) along with (14) and (15) into Equation (76), one deduces the solutions of Equation (74) as:

(I) Bright soliton solutions

$$\Phi(x,t)=\left\{2\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{c_3}}\operatorname{sech}\left[3\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right\}^{\frac{1}{3}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2t\right]}, \quad (80)$$

provided $c_3[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]<0$ and $(a-bv+\gamma)[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]<0$.

(II) Singular soliton solutions

$$\Phi(x,t)=\left\{2\sqrt{\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{c_3}}\operatorname{csch}\left[3\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right\}^{\frac{1}{3}}e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2t\right]}, \quad (81)$$

provided $c_3[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]>0$ and $(a-bv+\gamma)[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa]<0$.

9. Triple-Power Law

To this end, the nonlinearity form of the triple-power law is specified by

$$F\left(g^2\right)=c_1g^{2n}+c_2g^{4n}+c_3g^{6n}, \quad (82)$$

where c_1, c_2, and c_3 are constants and $c_3\neq 0$. Equation (1) using (82) becomes

$$i\Phi_t+a\Phi_{xx}+b\Phi_{xt}+\left(c_1|\Phi|^{2n}+c_2|\Phi|^{4n}+c_3|\Phi|^{6n}\right)\Phi+\gamma\left(\frac{|\Phi|_{xx}}{|\Phi|}\right)\Phi+\sigma(\Phi-ib\Phi_x)\frac{dW(t)}{dt}=i\delta\Phi_x, \quad (83)$$

Thus, Equation (4) takes the form

$$(a-bv+\gamma)g''+\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]g+c_1g^{2n+1}+c_2g^{4n+1}+c_3g^{6n+1}=0. \quad (84)$$

By using (13), we balancing g'' and g^{7n+1} in Equation (84), to derive $N = \frac{1}{n}$. Since N is not integer, one then takes

$$g(z) = [\varphi(z)]^{\frac{1}{n}}, \tag{85}$$

as long as $\varphi(z) > 0$. Inserting (85) into Equation (84) yields

$$(a - bv + \gamma)\Big[n\varphi\varphi'' + (1-n)\varphi'^2\Big] + n^2\Big[\Big(\omega - \sigma^2\Big)(b\kappa - 1) - a\kappa^2 - \delta\kappa\Big]\varphi^2 + n^2 c_1\varphi^4 + n^2 c_2\varphi^6 + n^2 c_6\varphi^8 = 0. \tag{86}$$

Now, we will employ the following method to solve Equation (86).

New Auxiliary Equation Approach

As a result, by using (13), we balance $\varphi\varphi''$ and φ^8 in Equation (86), to get $N = 1$. Consequently, from (11), the solution of Equation (86) has the same form (31). Substituting (31) and (12) with $M = 8$ into Equation (86), one derives the next algebraic equations,

$$\left.\begin{array}{l}
(a - bv + \gamma)H_1^2 r_8 + n^2 H_1^8 c_3 + 3(a - bv + \gamma)H_1^2 n r_8 = 0,\\[2ex]
\big[8H_1 n H_0 r_8 + H_1^2 r_7(1+5n)\big](a - bv + \gamma) + 16n^2 H_0 H_1^7 c_3 = 0,\\[2ex]
2n^2 H_1^6 c_2 + 56n^2 H_0^2 c_3 H_1^6 + \big[7H_1 n H_0 r_7 + 2(2n+1)H_1^2 r_6\big](a - bv + \gamma) = 0,\\[2ex]
(a - bv + \gamma)\big[(3n+2)H_1^2 r_5 + 6H_1 n H_0 r_6\big] + 56n^2 H_0^3 c_3 H_1^5 + 6n^2 H_0 H_1^5 c_2 = 0,\\[2ex]
2\big(28H_0^4 c_3 + 6c_1 + 15H_0^2 c_2\big)n^2 H_0^2 H_1^2 + \big(3H_1 n H_0 r_3 + 2H_1^2 r_2\big)(a - bv + \gamma)\\
+2n^2 H_1^2\big[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa\big] = 0,\\[2ex]
\big[4H_1 n H_0 r_4 + (n+2)H_1^2 r_3\big](a - bv + \gamma) + 8n^2 H_1^3\big(5H_0^3 c_2 + H_0 c_1 + 14H_0^5 c_3\big) = 0,\\[2ex]
(a - bv + \gamma)\big[5H_1 n H_0 r_5 + 2H_1^2 r_4(n+2)\big] + 2n^2 H_1^4\big(c_1 + 15H_0^2 c_2 H_1^4 + 70H_0^4 c_3\big) = 0,\\[2ex]
\big[2H_1^2 r_0(1-n) + H_1 n H_0 r_1\big](a - bv + \gamma) + 2n^2\big(H_0^8 c_3 + H_0^6 c_2 + H_0^4 c_1\big)\\
+2n^2 H_0^2\big[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa\big] = 0,\\[2ex]
[2nH_0 r_2 + H_1 r_1(2-n)](a - bv + \gamma) + 4n^2\big(4H_0^7 c_3 + 2H_0^3 c_1 + 3H_0^5 c_2\big)\\
+4n^2 H_0\big[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa\big] = 0
\end{array}\right\} \tag{87}$$

Thus, set $r_0 = r_1 = r_3 = r_4 = r_6 = r_7 = 0$, in Equation (87) and solving them by using the Maple, one obtains

$$H_0 = 0,\ H_1 = \left[-\frac{(1+3n)(a-bv+\gamma)r_8}{n^2 c_3}\right]^{\frac{1}{6}},\ r_2 = -\frac{n^2\big[(\omega-\sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa\big]}{a - bv + \gamma},\ r_5 = 0,\ r_8 = r_8, \tag{88}$$

and

$$c_1 = 0,\ \ c_2 = 0, \tag{89}$$

provided $c_3(a - bv + \gamma)r_8 < 0$. Consequently, inserting (88) along with (14) and (15) into Equation (31), one deduces the solutions of Equation (83) as:
(I) Bright soliton solutions

$$\Phi(x,t) = \left\{\sqrt{-\frac{(3n+1)\big[(\omega-\sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa\big]}{c_3}}\operatorname{sech}\left[3n\sqrt{-\frac{(\omega-\sigma^2)(b\kappa-1) - a\kappa^2 - \delta\kappa}{a - bv + \gamma}}(x - vt)\right]\right\}^{\frac{1}{3n}} e^{i[-\kappa x + \omega t + \sigma W(t) - \sigma^2 t]}, \tag{90}$$

provided $c_3[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] < 0$ and $(a - bv + \gamma)[(\omega - \sigma^2)(b\kappa - 1) - a\kappa^2 - \delta\kappa] < 0$.
(II) Singular soliton solutions

$$\Phi(x,t) = \left\{ \sqrt{\frac{(3n+1)\left[\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa\right]}{c_3}} \operatorname{csch}\left[3n\sqrt{-\frac{\left(\omega-\sigma^2\right)(b\kappa-1)-a\kappa^2-\delta\kappa}{a-bv+\gamma}}(x-vt)\right]\right\}^{\frac{1}{3n}} e^{i\left[-\kappa x+\omega t+\sigma W(t)-\sigma^2 t\right]}, \tag{91}$$

provided $c_3[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa] > 0$ and $(a-bv+\gamma)[(\omega-\sigma^2)(b\kappa-1)-a\kappa^2-\delta\kappa] < 0$.

10. Conclusions

In this article, we found soliton solutions for the stochastic resonant NLSE (1) with the spatio-temporal dispersion and inter-modal dispersion having multiplicative white noise in the Itô sense. Our study is concentrated on the functional $F\left(|\Phi|^2\right)$, which takes seven nonlinear forms, via Kerr law, power law, parabolic law, dual-power law, quadratic–cubic law, polynomial law, and triple-power law. We have applied the new auxiliary equation method to find the bright, dark, and singular soliton solutions of Equation (1) for these seven nonlinear forms. Certain parameter constraints are involved to ensure the existence of such solutions. The stochastic soliton solutions obtained in this article are accurate and important in understanding physical phenomena. The effect of multiplicative noise on these solutions has been illustrated using some graphical representations (see Figures 1–4). Finally, our work is new and has a lot of openings that would lead to an abundance of new results which are yet to be explored.

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