

Article

Construction of a Dimensional Damage Model of Reinforced Concrete Columns under Explosion Loading

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Abstract: When studying the damage law of reinforced concrete building structures under explosive loading, the direct experimental cost is too high and numerical simulations take a long time. Based on the theoretical analysis, the dimensional analysis model of reinforced concrete members under explosive loading can be used to study the damage law of reinforced concrete members under explosive loading. It provides guidance, reduces the number of tests, improves the efficiency of the test, and has certain research significance. In this paper, a typical reinforced concrete column is taken as the main research object. Based on the dimensional analysis method, the relationship between the damage to the reinforced concrete column and the explosion equivalent and explosion distance under explosion loading is studied. The finite element simulation software LS-DYNA 18.2 is used to determine the function relationship between the disturbance in the column and the proportional explosion distance. The results show that when the proportional explosion distance Z is greater than $0.0693 \text{ m/kg}^{1/3}$, the center disturbance of the blasting surface of the reinforced concrete column has a linear relationship with the reciprocal of the proportional explosion distance. The linear relationship can be used to predict the column's center disturbance under partial explosion conditions, which provides guidance for studying the damage criterion of reinforced concrete under explosion loading.

Keywords: reinforced concrete column; dimensional analysis method; explosion load; proportional explosive distance



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1. Introduction

Typical reinforced concrete building structures are generally composed of plates, beams, columns and other components. Among them, reinforced concrete columns, as one of the main load-bearing components, are likely to be seriously damaged or even cause continuous collapse of the structure under the impact of certain explosion loads, which eventually leads to complete damage to the structure [1,2].

Based on this background, a large number of scholars have carried out research on the damage effect of reinforced concrete columns. Jiao [2] studied the dynamic response of reinforced concrete columns under explosion loading under two kinds of constraints and obtained the variation law of lateral displacement of reinforced concrete columns under explosion impact loads at a reduced distance. Wei et al. [3] studied the dynamic response process of reinforced concrete columns fixed at both ends under explosion loads and analyzed the changes in lateral displacement and the failure of columns under different reduced distances. The results show that when the reduced distance is about 2.0, the influence of the explosion load on the column can be neglected. Su et al. [4] directly quoted the concept of reduced distance and proposed a relationship between the lateral displacement of reinforced concrete columns and the reduced distance by means of numerical simulation. Peng [5] carried out numerical simulations under different proportional distances and

explosive equivalents and studied the dynamic response and damage law of reinforced concrete columns impacted by explosive loads when the explosive was detonated on the ground. It shows that the damage response of reinforced concrete columns is particularly sensitive to the proportional distance under the impact of an explosion load. Yan et al. [6] studied the influence of different reinforcement ratios and axial compression ratios on the damage to concrete columns through the combination of numerical simulations and experimental research. Wu et al. [1] analyzed the influence of five damage factors (explosive equivalent, stirrup reinforcement ratio, concrete strength, explosion point distance, longitudinal reinforcement ratio, etc.) on the damage to columns using numerical simulations of reinforced concrete columns under different explosion conditions. Based on finite element simulation analysis software such as ANSYS/LS-DYNA, Xiao et al. [7] studied the damage to the building structure after different explosions with different explosion positions and ductilities of the building's frame structure. The analysis shows that the initiation point of the explosive is the most sensitive factor affecting the damage to the frame structure. Chen et al. [8] studied the dynamic response of reinforced concrete columns in near explosions through experimental research and numerical simulations and analyzed the influence of various factors (reinforcement ratio, proportional distance, length–diameter ratio, etc.) on the damage to columns under explosion loading. He et al. [9,10] carried out a numerical simulation of the damage to reinforced concrete slabs and reinforced concrete beams under different proportional distances and proposed a prediction formula with the peak displacement of the mid-span as the response factor, which provides some help for studying the damage to slabs under different explosion conditions. Wang et al. [11] observed the failure modes and failure characteristics of three-sided supported reinforced concrete slabs under different working conditions by conducting experiments on the three-sided supported reinforced concrete slabs at different explosion distances and charge quantities, summarizing the influence of the proportional explosion distance on the damage to the slabs. Chen [12] studied the dynamic response (including various stresses and displacements) of reinforced concrete T-beams reinforced with carbon fiber composite materials under different explosion loads by numerical simulations and summarized the influence of different parameters on their anti-explosion performance. Cui [13] carried out experiments and numerical simulations of the damage to composite hollow-concrete-filled steel tubular columns under explosive loading. The influence of various parameters of the column on the interaction between the explosive shock wave and the structural column, in addition to the law of the explosive load acting on the column, was studied. The influence of the proportional distance on damage to the column was put forward, with the peak displacement of the column as the evaluation standard. Based on finite element simulation software, Li [14] studied the relationship between two damage indexes (i.e., lateral residual displacement and vertical residual bearing capacity) under the influence of different factors and fitted the damage degree index of vertical bearing capacity under different variables. The formulas of the longitudinal reinforcement ratio, section width and lateral residual displacement can be used to evaluate the damage degree of reinforced concrete columns at the scene of an explosion accident.

Based on a theoretical analysis and numerical simulations, Park et al. [15] studied the variation in anti-explosion performance of reinforced concrete columns with the axial force and slenderness ratio and provided a reference for the evaluation of the damage degree of reinforced concrete columns under explosion loading. Morrill et al. [16] obtained a large number of original test data from the field through testing the equal proportion of reinforced concrete columns under explosion loads under different working conditions, which provided a theoretical basis for subsequent scholars to carry out numerical simulation analysis. Bao et al. [17] proposed a simple analysis model to evaluate the compressive arching effect of reinforced concrete beams under the middle column removal scenario. The model considers more comprehensive factors and has a good accuracy in predicting the bearing capacity, displacement and maximum beam compression force of compressive arches. Vanni et al. [18] proposed a graphical and easy-to-use tool to determine the beam–

column joint sizes available at the initial stage of the reinforced concrete building design process. At the same time, the paper presents a nomogram related to the reinforcement ratio coefficient including seven different concrete grades and covering most of the joints, which is convenient for users to customize the nomogram according to their own needs. Ma et al. [19] carried out quasi-static large displacement tests under different demolition schemes on the same reinforced concrete slab substructure and studied the influence of different demolition schemes on the peak pressure, ultimate bearing capacity and displacement of the column.

In summary, some scholars have studied the influence of column end constraint conditions on the damage to reinforced concrete column structures under blast impacts, and the scaled distance Z has been introduced to propose a critical reduced distance. This shows that when the reduced distance reaches a critical value, the column is less affected by the blast load; a large number of scholars, led by Peng Liying [5], have studied the influence of various factors on reinforced concrete columns under explosive loading and analyzed the influence of various factors on the damage to the column. No scholars have put forward a more detailed relationship between the explosion factor and the damage to the column.

Under explosion loading, the damage degree of reinforced concrete column structures can be characterized by the column disturbance before the explosion [20–27]. In this paper, the dimensional analysis method is used to study the relationship between the maximum disturbance of a reinforced concrete column under explosive loading and the explosion equivalent and the initiation distance, and a theoretical model function is obtained. The finite element simulation software LS-DYNA was used to carry out the numerical simulations of the damage to the column under different explosion equivalents and explosion distances. By analyzing the variation in the maximum disturbance of the column with these two variables, the coefficients in the theoretical model function were fitted. The theoretical model can provide a simplified research method for subsequent scholars to study the explosion damage to reinforced concrete columns and can also analyze the explosion conditions according to the damage to reinforced concrete columns after an explosion.

2. II Theorem of Dimensional Analysis

2.1. Theory [28]

The dimensional analysis method is the most commonly used method to analyze complex phenomena for which the mechanism is unclear and the governing laws are not fully understood in order to obtain the similarity criterion, which is widely used in modern engineering and technical problems.

The basic theorem of dimensional analysis is the II theorem. It is based on the fact that any physical process contains n -dimensional physical quantities. If n_1 physical quantities are selected as the basic physical quantities, this physical process can be described by the relationship of n_1 dimensionless quantities composed of n physical quantities, where n and n_1 are arbitrary real numbers, and $n > n_1$. Since Π_i (i is a real number, generally $1, 2, \dots, n - n_1$) is used to represent these dimensionless quantities, it is called the II theorem.

2.2. Cylinder Damage Model Based on the II Theorem

Figure 1 shows the damage model of reinforced concrete support columns under blast loading, including reinforced concrete columns (longitudinal bars, stirrups and concrete), spherical explosives, rigid walls and air domains.

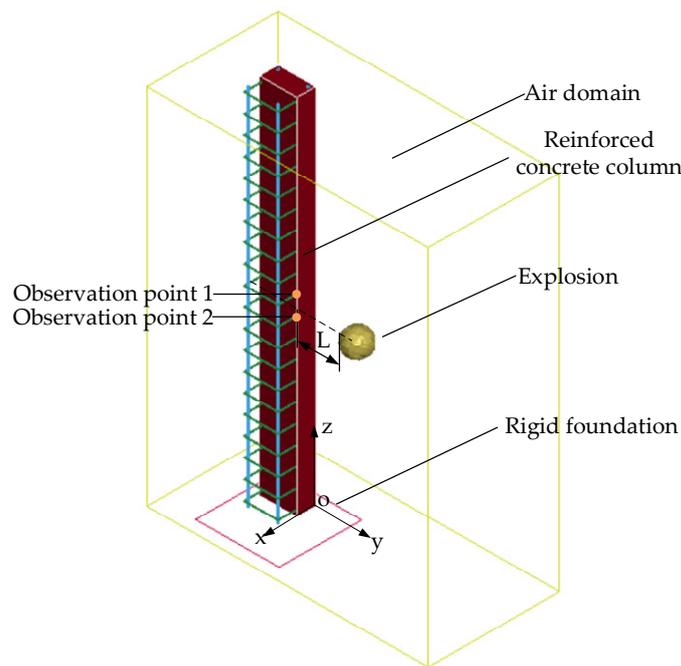


Figure 1. Damage diagram of a reinforced concrete column.

The symbolic interpretation of the physical parameters in the damage model of reinforced concrete columns is shown in Table 1.

Table 1. Interpretation table of physical parameters in damage model.

Symbol	Unit	Explanation
m	kg	Explosive equivalent
r	m	Radius of spherical explosive
L	m	The minimum distance from the detonation point of explosive to the surface of reinforced concrete column
ρ_{TNT}	kg/m ³	Explosive density
ρ	kg/m ³	Column density
H	m	Column height
a	m	The section (square) side length of the column
σ	Pa	The yield strength of the column
E	Pa	Young's modulus of column
I	m ⁴	Sectional moment of inertia of the column
b	m ²	On the cross-section of the column, the total cross-sectional area of the longitudinal reinforcement
ξ	m	Disturbance in column

The basic physical parameters involved in the damage model of reinforced concrete columns are mass M , length L and time T ; this paper selects the 'MLT' dimensional analysis system to analyze the damage model. If it is determined that the parameters in Table 1 are the influencing factors of cylinder damage and the degree of disturbance in the column is used as the basis for evaluating the degree of cylinder damage, then the physical problem can be described as follows according to the principle of dimensional analysis:

$$\xi = f(m, r, L, H, a, b, \rho_{TNT}, \rho, \sigma, E, I) \tag{1}$$

The bending stiffness of the cylinder is characterized by EI , which is the combination of Young's modulus E and the cross-section moment of inertia I . At the same time, the

density of spherical explosive ρ_{TNT} can be expressed by explosive equivalent m and radius r . Then, the above equation can be simplified to:

$$\zeta = f(m, r, L, H, a, b, \rho, \sigma, EI) \tag{2}$$

Equation (2) has a total of 10 change parameters, including 1 dependent variable parameter and 9 independent variable parameters, and its dimensional power exponent is shown in Table 2.

Table 2. Dimension power exponent table of physical quantities.

	ζ	m	r	L	H	a	b	ρ	σ	EI
M	0	1	0	0	0	0	0	1	1	1
L	1	0	1	1	1	1	2	-3	-1	3
T	0	0	0	0	0	0	0	0	-2	-2

The density ρ , column height H and yield strength σ of concrete columns are selected as reference physical quantities, and Table 2 is sorted and reconfigured. Table 3 can be obtained as follows.

Table 3. Dimension power exponent table of physical quantities (after sorting).

	ρ	H	σ	m	r	L	a	b	EI	ζ
M	1	0	1	1	0	0	0	0	1	0
L	-3	1	-1	0	1	1	1	2	3	1
T	0	0	-2	0	0	0	0	0	-2	0

Table 4 can be obtained by transforming Table 3.

Table 4. Dimension power exponent table of physical quantities (after transformation).

	ρ	H	σ	m	r	L	a	b	EI	ζ
ρ	1	0	0	1	0	0	0	0	0	0
H	0	1	0	3	1	1	1	2	4	3
σ	0	0	1	0	0	0	0	0	1	0

According to the properties of Π theorem and dimensional analysis, six dimensionless independent variables and one dimensionless dependent variable can be obtained, see Formula (3). In this formula, dimensionless independent variables are represented by π_i , and the dimensionless dependent variable is represented by π .

$$\left\{ \begin{array}{l} \pi_1 = \frac{m}{\rho H^3} \\ \pi_2 = \frac{r}{H} \\ \pi_3 = \frac{L}{H} \\ \pi_4 = \frac{a}{H} \\ \pi_5 = \frac{b}{H^2} \\ \pi_6 = \frac{EI}{\sigma H^4} \\ \pi = \frac{\zeta}{H} \end{array} \right. \tag{3}$$

Therefore, Formula (3) can be written as a dimensionless function expression:

$$\frac{\zeta}{H} = f\left(\frac{m}{\rho H^3}, \frac{r}{H}, \frac{L}{H}, \frac{a}{H}, \frac{b}{H^2}, \frac{EI}{\sigma H^4}\right) \tag{4}$$

Assuming that the size of the cylinder and the relevant physical parameters do not change and only the explosive equivalent and the detonation distance change, the above equation can be simplified to:

$$\frac{\zeta}{H} = f\left(\frac{m}{\rho H^3}, \frac{L}{H}\right) \tag{5}$$

In general, when the explosive equivalent is constant, the greater the detonation distance, the smaller the damage degree of the column and the smaller the disturbance in the column. Therefore, the independent variable L/H in the above formula can be adjusted to H/L , and the above formula can be reduced to:

$$\frac{\zeta}{H} = f\left(\frac{m}{\rho H^3}, \frac{H}{L}\right) \tag{6}$$

It can be seen that ζ/H in Formula (4) is related to the independent variables $m/(\rho H^3)$ and H/L , but the specific functional relationship is not clear. In order to make the function relationship more concise and clearer, the function relationship of the variable separation method in mathematics is analyzed. Therefore, the two independent variables are coupled exponentially, and the new dependent variable is represented by D_m , which is expressed as follows:

$$\frac{\zeta}{H} = \varphi(D_m) = \varphi\left[\left(\frac{m}{\rho H^3}\right)^\alpha \left(\frac{H}{L}\right)^\beta\right] \tag{7}$$

where α and β are undetermined coefficients. In order to obtain the values of α and β in the above expressions, a damage model of reinforced concrete column is established and a numerical simulation is carried out. The undetermined coefficients are obtained by analyzing the simulation conditions.

3. Numerical Simulations to Obtain the Undetermined Coefficient

3.1. Verification of the Numerical Simulation Method

In order to verify the reliability of the simulation model, the near-explosion experimental results of Yan et al. [5] are compared with the simulation results. In the experiment of Yan et al. [5], a 1/2 scaled model of a reinforced concrete column was established, and the damage effect of the column under three explosion conditions was studied. Finite element simulation software was used to reproduce the experiments in their paper. A numerical simulation of the damage to the column under three working conditions was carried out. The curve of the maximum displacement in the column with time under different working conditions was obtained (see Figure 2). The maximum displacement in the column is used as the comparison standard. The experimental results and simulation results are listed in Table 5, and an error analysis was carried out.

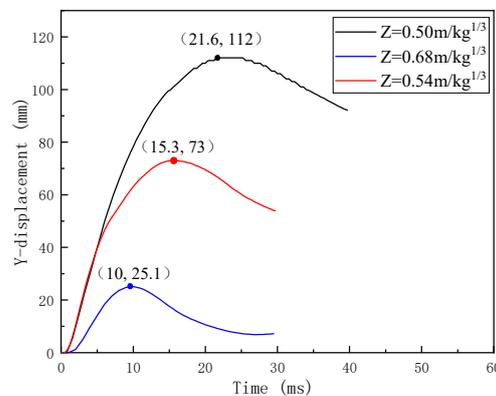


Figure 2. Simulation result curve.

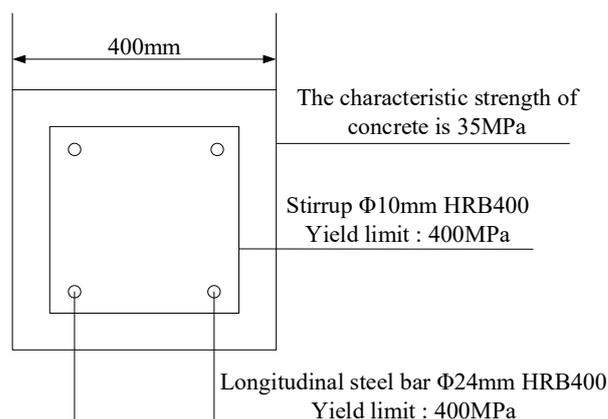
Table 5. Validation model results comparison table.

Experiment Number	Z (m/kg ^{1/3})	m (kg)	Hm (mm)	Maximum Displacement in Column (mm)	Simulation Result (mm)	Error
B1-1	0.5	1	500	97.2	112	15.2%
B1-2	0.68	0.4	500	29.4	25.1	14.6%
B1-3	0.54	0.8	500	64.5	73	13.2%

Because the maximum error between the simulation results and the experimental results of the reference paper is 15.2%, which is within the allowable range of error, it is proven that the modeling method and the simulation model have certain correctness.

3.2. Establishment of a Finite Element Simulation Model

By consulting the literature [29], this paper establishes a reinforced concrete column, its size is as follows: the section size of reinforced concrete column is 400 mm × 400 mm and the height is 3900 mm. The diameter of longitudinal reinforcement is 24 mm, the diameter of the stirrup is 10 mm, the spacing of stirrup is 200 mm, and the thickness of concrete cover is 20 mm. Figure 3 is the reinforcement diagram of the reinforced concrete column.

**Figure 3.** Reinforcement diagram of the reinforced concrete column.

This paper mainly uses ANSYS/LS-DYNA finite element simulation software for numerical simulations. The S-ALE algorithm is mainly used to establish a ‘concrete–explosive–air’ fluid–solid coupling model to accurately observe the damage to reinforced concrete under the impact of explosion loads. Among them, the reinforced concrete column adopts the non-common node separation modeling method, and the keyword *CONSTRAINEDB_BEAM_IN_SOLID is used to constrain the coupling relationship between the steel bar and the concrete column. A rigid wall is set at the bottom of the concrete column to simulate the actual ground, adding a non-reflective boundary at the boundary of the air domain and applying gravity to the building. The spherical TNT charge is selected as the explosive, and the keywords *MAT_HIGH_EXPLOSIVE_BURN and *EOS_JWL are used to describe the material model and state equation of the explosive. Air uses the keywords *MAT_NULL and *EOS_LINEAR_POLYNOMIAL and to describe its material model and state equation [30,31]. The grid size is as follows: a 2 cm grid element is selected for reinforced concrete columns and a 3 cm grid element is selected for the air domain. Figure 4 is the grid model of the reinforced concrete column.

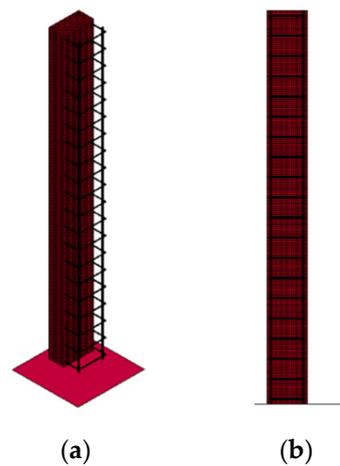


Figure 4. Mesh model of the reinforced concrete column. (a) is a 45° view of the cylinder; (b) is the side view of the cylinder.

3.3. Numerical Simulation Conditions

Five kinds of explosive equivalents (1 kg, 2 kg, 3 kg, 4 kg, 5 kg) and five kinds of explosion distances (0.1 m, 0.2 m, 0.3 m, 0.4 m, 0.5 m) were selected to simulate twenty-five kinds of explosion conditions to determine the undetermined coefficients in Formula (7). The specific simulation conditions are shown in Table 6.

Table 6. Numerical simulation conditions.

Simulation of Condition	Explosive Equivalent m (kg)	Initiation Distance L (m)	Simulation of Condition	Explosive Equivalent m (kg)	Initiation Distance L (m)
Z1-1	1	0.1	Z4-1	4	0.1
Z1-2		0.2	Z4-2		0.2
Z1-3		0.3	Z4-3		0.3
Z1-4		0.4	Z4-4		0.4
Z1-5		0.5	Z4-5		0.5
Z2-1	2	0.1	Z5-1	5	0.1
Z2-2		0.2	Z5-2		0.2
Z2-3		0.3	Z5-3		0.3
Z2-4		0.4	Z5-4		0.4
Z2-5		0.5	Z5-5		0.5
Z3-1	3	0.1			
Z3-2		0.2			
Z3-3		0.3			
Z3-4		0.4			
Z3-5		0.5			

Through the numerical simulation of the above 25 explosion conditions, the simulation data are recorded.

3.4. Numerical Simulation Results and Analysis

Due to the explosive loading, the concrete on the blasting surface of reinforced concrete easily fails, but the stirrup does not easily fail. Therefore, the average displacement of the two observation points on the stirrup at the center is used to characterize the disturbance in the column. The two observation points are recorded as observation point 1 and observation point 2 (see Figure 1). The coordinate system is established by taking an end point at the bottom of the column as a dot and the coordinates of observation point 1 and observation point 2 are (20, -2, 185) and (20, -2, 205). The numerical simulation results are shown in Table 7.

Table 7. Numerical simulation results table.

Simulation Condition	Maximum Displacement in Column (cm)			Simulation of Condition	Maximum Displacement in Column (cm)		
	Observation Point 1	Observation Point 2	Average Value		Observation Point 1	Observation Point 2	Average Value
Z1-1	0.645	0.673	0.659	Z3-4	0.423	0.452	0.4375
Z1-2	0.278	0.285	0.2815	Z3-5	0.249	0.259	0.254
Z1-3	0.139	0.145	0.142	Z4-1	Stirrup fracture scattered		
Z1-4	0.068	0.069	0.0685	Z4-2	2.010	2.370	2.190
Z1-5	0.021	0.022	0.0215	Z4-3	1.230	1.280	1.255
Z2-1	0.149	1.92	1.0345	Z4-4	0.733	0.775	0.754
Z2-2	0.628	0.651	0.6395	Z4-5	0.433	0.458	0.4455
Z2-3	0.423	0.441	0.432	Z5-1	Stirrup fracture scattered		
Z2-4	0.198	0.203	0.2005	Z5-2	3.070	3.660	3.365
Z2-5	0.120	0.126	0.123	Z5-3	1.740	1.790	1.765
Z3-1	3.550	4.320	3.935	Z5-4	1.060	1.100	1.080
Z3-2	1.400	1.510	1.455	Z5-5	0.658	0.689	0.6735
Z3-3	0.814	0.838	0.826				

When the explosion distance is 0.1 m, when the 4 kg and 5 kg explosion equivalents are detonated, the large-area failure of the concrete at the center of the blasting surface column disappears, and the stirrup breaks and disperses (its damage diagram is shown in Figure 5). This time, the column is seriously damaged and it is difficult to use the disturbance in the column to study the damage to reinforced concrete columns. Therefore, when using software to analyze the numerical simulation data, Z4-1 and Z5-1 conditions were not considered.

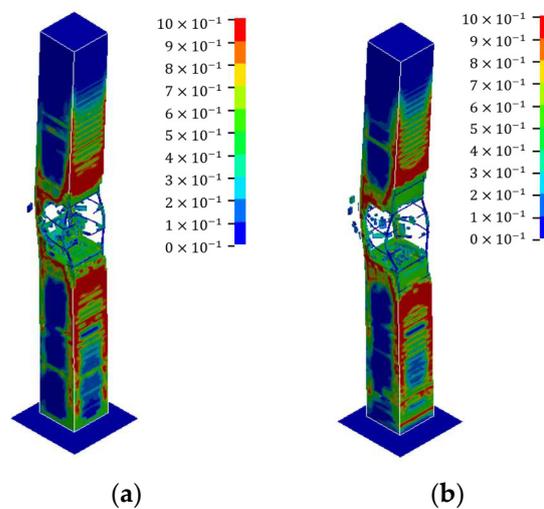


Figure 5. Damage diagram of the reinforced concrete column. (a) is the damage diagram for the Z4-1 condition; (b) is the damage diagram for the Z5-1 condition.

Using Origin software to fit and analyze the simulation results under the same explosion equivalent and the same explosion distance, Figures 6 and 7 were obtained. Figure 6 shows the relationship between the disturbance ξ in the column and the explosion distance and the explosion equivalent under different explosion equivalents. Figure 7 shows the relationship between the disturbance ξ in the column and the explosion distance and explosion equivalent under different explosion distances.

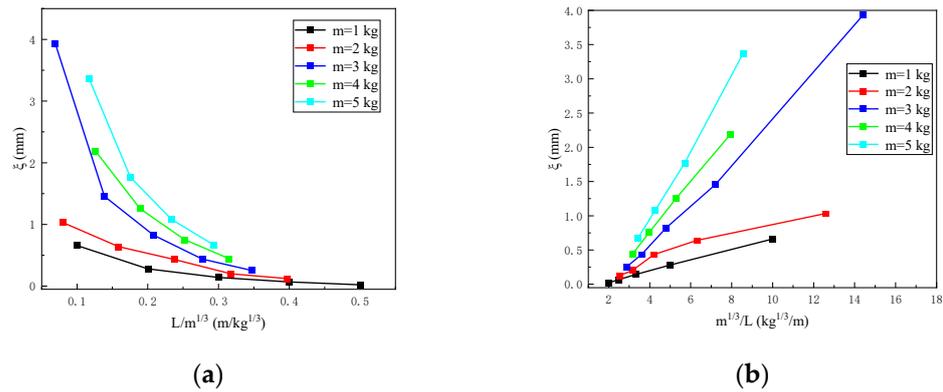


Figure 6. The relationship between the disturbance and the explosion distance and the explosion equivalent in the column under different explosion equivalents. (a) is the change curve of ξ and $L/m^{1/3}$; (b) is the change curve of ξ and $m^{1/3}/L$.

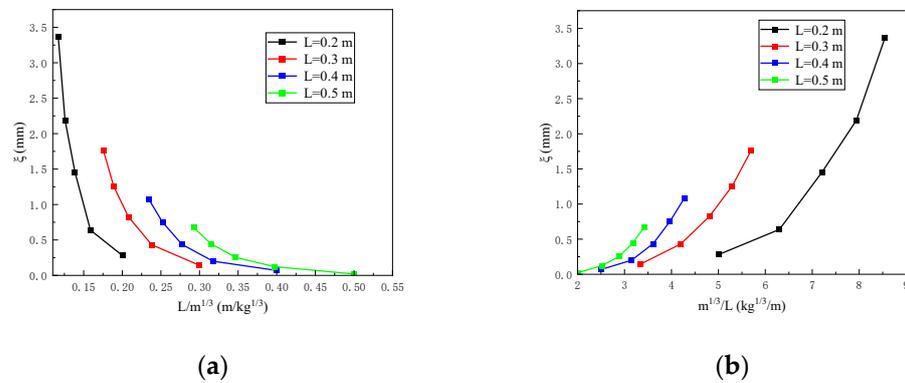


Figure 7. The relationship between the disturbance in the column and the explosion distance and the explosion equivalent under different explosion distances. (a) is the change curve of ξ and $L/m^{1/3}$; (b) is the change curve of ξ and $m^{1/3}/L$.

The proportional detonation distance Z is introduced, and the proportional detonation distance is defined as the ratio of the detonation distance to the third power of the explosion equivalent, that is,

$$Z = L/m^{1/3} \tag{8}$$

In Figures 6a and 7a, the curve abscissa can be expressed as Z , and in Figures 6b and 7b, the curve abscissa can be expressed as $1/Z$. By observing the five curves in Figure 6a, it can be found that under the same explosion equivalent, the disturbance in the column decreases with the increase in the proportional explosion distance Z in a similar and nonlinear manner. In Figure 6b, after modifying the abscissa of the curve to $1/Z$, it can be clearly observed that under the same explosive equivalent, the linear relationship between the disturbance ξ in the reinforced concrete column and $1/Z$ is generally linear, and the greater the explosive equivalent, the greater the slope of the linear line fitted by the disturbance ξ and $1/Z$ in the column. Without considering the simulation conditions of Z4-1 and Z5-1, the data are less when $L = 0.1$ m. Therefore, the relationship between the disturbance ξ in the column and the proportional detonation distance Z is not analyzed when $L = 0.1$ m. By observing the four curves in Figure 7a, it can be found that under different explosion distances, the disturbance in the column decreases similarly and nonlinearly with the increase in the proportional explosion distance Z ; in Figure 7b, after adjusting the abscissa of the curve to $1/Z$, it can be observed that the nonlinear relationship between the disturbance ξ and $1/Z$ in the reinforced concrete column still shows a similar change trend under the same explosion distance.

According to Figure 6b, it can be seen that the five curves have obvious linear relationships, and the function model obtained by linear fitting is shown in Table 8.

Table 8. Linear fitting results under different explosion equivalents.

Explosive Equivalent m	Linear Fitting Curve Diagram	Fitting Equation
1 kg		$\xi = 0.079\frac{1}{Z} - 0.13$
2 kg		$\xi = 0.087\frac{1}{Z} - 0.017$
3 kg		$\xi = 0.32\frac{1}{Z} - 0.73$
4 kg		$\xi = 0.36\frac{1}{Z} - 0.7$

Table 8. Cont.

Explosive Equivalent m	Linear Fitting Curve Diagram	Fitting Equation
5 kg		$\zeta = 0.53\frac{1}{Z} - 1.2$

Based on the five curves obtained by linear fitting, the mean square error (MSE) can be used to evaluate the change in the data. The mean square error refers to the average value of the square of the distance between the predicted value $f(x)$ of the model and the real value y of the sample [32]. The formula is shown in Formula (9).

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - f(x_i))^2 \tag{9}$$

where y_i and $f(x_i)$ represent the true value and predicted value of the i th sample, respectively, and n is the number of samples. The closer the mean square error MSE is to 0, the smaller the fitting error is.

The mean square error values of the five linear fitting functions are 6.48×10^{-5} , 6×10^{-3} , 4.3×10^{-3} , 1.96×10^{-4} and 1.8×10^{-3} , respectively. The maximum mean square error value is 6×10^{-3} , which is close to 0, which proves that the fitting error of the five linear fitting curves is low and the fitting effect is good. That is to say, it is proven that there is a certain linear relationship between the perturbation ζ in the column and the reciprocal of the proportional explosion distance Z under the same explosion equivalent. Thus, the α value in Formula (7) is 1/3 and the β value is 1. Formula (7) can be simplified to

$$\frac{\zeta}{H} = \varphi(D_m) = \varphi\left[\left(\frac{m}{\rho H^3}\right)^{\frac{1}{3}} \left(\frac{H}{L}\right)^1\right] = \varphi\left[\rho^{\frac{1}{3}} \frac{1}{Z}\right] \tag{10}$$

That is:

$$\zeta = H\varphi\left[\rho^{\frac{1}{3}} \frac{1}{Z}\right] \tag{11}$$

Because, in this numerical simulation, the structural size of the reinforced concrete column does not change, only the explosion distance and the explosion equivalent change, that is, H and ρ in Formula (11) are fixed. Therefore, Formula (11) can be understood as when the structural size of the reinforced concrete column is constant. Under any explosion equivalent, the disturbance ζ in the column is only linearly related to the reciprocal of the proportional explosion distance Z , that is:

$$\zeta = P\frac{1}{Z} + Q \tag{12}$$

In this formula, P and Q are constant coefficients, and $P > 0$.

After Formula (12) is obtained, the numerical simulation under the same explosion equivalent and two explosion distances can be carried out, and the functional relationship between the deflection ζ in the column and the proportional explosion distance Z under

a certain explosion equivalent can be determined, which can be used to infer the column disturbance at any distance under the explosion equivalent.

Before the column is seriously damaged (the penetrating hole appears in the center of the reinforced concrete column and the stirrup breaks and disperses), the column damage can be evaluated by the column perturbation. The proportional detonation distances of 25 numerical simulation conditions in Table 7 are calculated, and the proportional detonation distances are sorted to obtain Table 9.

Table 9. Sorting table of proportional burst distance.

Simulation of Condition	Explosive Equivalent m (kg)	Initiation Distance L (m)	Proportional Detonation Distance Z (m/kg ^{1/3})	Scheduling
Z1-1	1	0.1	0.1000	5
Z1-2		0.2	0.2000	12
Z1-3		0.3	0.3000	19
Z1-4		0.4	0.4000	24
Z1-5		0.5	0.5000	25
Z2-1	2	0.1	0.0794	4
Z2-2		0.2	0.1587	9
Z2-3		0.3	0.2381	15
Z2-4		0.4	0.3175	21
Z2-5		0.5	0.3969	23
Z3-1	3	0.1	0.0693	3
Z3-2		0.2	0.1387	8
Z3-3		0.3	0.2080	13
Z3-4		0.4	0.2773	17
Z3-5		0.5	0.3467	22
Z4-1	4	0.1	0.0630	2
Z4-2		0.2	0.1260	7
Z4-3		0.3	0.1890	11
Z4-4		0.4	0.2520	16
Z4-5		0.5	0.3150	20
Z5-1	5	0.1	0.0585	1
Z5-2		0.2	0.1170	6
Z5-3		0.3	0.1754	10
Z5-4		0.4	0.2339	14
Z5-5		0.5	0.2924	18

Among the 25 working conditions of this numerical simulation, the damage to reinforced concrete columns is serious only in Z4-1 and Z5-1, and the other 23 working conditions can be used to evaluate the damage to the columns. According to Table 8, it can be found that the proportional detonation distance under Z4-1 and Z5-1 conditions is the smallest. Therefore, among the 25 working conditions, the third proportional detonation distance is 0.0693 m/kg^{1/3} under the Z3-1 condition. When the proportional detonation distance Z is greater than 0.0693 m/kg^{1/3}, Equation (12) can be used to calculate the column perturbation.

3.5. Validation of Linear Relationship

The test results and calculation results of reinforced concrete members in Wang's paper [33] and He's paper [10] are selected for a comparative analysis. Table 10 is the peak displacement results of reinforced concrete members in the reference paper.

Table 10. Results from reference papers of the peak displacement in reinforced concrete members.

Data Sources	Order Number	Proportional Detonation Distance Z (m/kg ^{1/3})	1/Z (kg ^{1/3} /m)	Center Shift (mm)
Reference [33]	1	0.57	1.75	9
Reference [33]	2	0.50	2	25
Reference [33]	3	0.44	2.27	35
Reference [33]	4	0.40	2.5	40
Reference [10]	5	0.4	2.5	41.09
Reference [10]	6	0.7	1.43	18.92
Reference [10]	7	1.0	1	8.55
Reference [10]	8	1.3	0.77	4.96
Reference [10]	9	1.6	0.63	2.75
Reference [10]	10	1.2	0.83	5.38

Two data points in paper [32] and paper [10] are selected, respectively, to determine the P and Q parameters in Formula (12), and $P_1 = 41.3, Q_1 = -63.25$ in prediction Formula (1) and $P_2 = 20.72, Q_2 = -10.71$ in prediction formula (2) can be obtained. Figure 8 was drawn, the literature data and the prediction curve were recorded, the prediction formula was used to predict the remaining data in the reference paper, and an error analysis was performed. Table 11 shows the data from the error analysis.

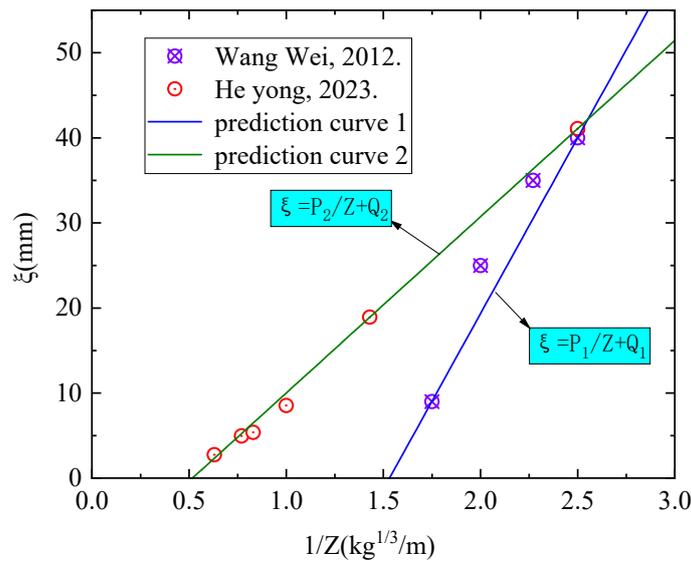


Figure 8. Comparison of experimental data and predicted data [10,33].

Table 11. Data error analysis table.

Data Sources	Data Sources	Order Number	Proportional Detonation Distance Z (m/kg ^{1/3})	Center Shift (mm)	Prediction Results (mm)	Error
Reference [33]	Reference [32]	1	0.57	9	9	0
Reference [33]	1	2	0.50	25	19.35	22.6%
Reference [33]	1	3	0.44	35	30.51	12.82%
Reference [33]	1	4	0.40	40	40	0
Reference [10]	2	5	0.40	41.09	41.9	0
Reference [10]	2	6	0.70	18.92	18.89	0
Reference [10]	2	7	1.00	8.55	10.01	17.08%
Reference [10]	2	8	1.30	4.96	5.23	5.4%
Reference [10]	2	9	1.60	2.75	2.24	-18.55%
Reference [10]	2	10	1.20	5.38	6.56	21.87%

Through calculation, the maximum error of the predicted value is 22.6%, which is not over 25%. Therefore, it is proven that the established linear relationship between the disturbance in the column and the reciprocal of the proportional detonation distance has certain reliability.

4. Conclusions

Most modern buildings have reinforced concrete structures, and reinforced concrete columns are one of the main load-bearing components. Damage under explosion loading will lead to serious damage or even collapse of the building structure. Therefore, it is urgently required to study the damage effect of reinforced concrete columns under explosion loads. However, the theoretical analysis of the explosion load is complicated, the experimental research cost is large and the numerical simulation time is long. It is particularly important to use dimensional analysis theory to guide the research on this problem. In this paper, the relationship between the disturbance in the reinforced concrete columns and the explosion equivalent and explosion distance under explosion loading is studied via a dimensional analysis method, and the dimensional analysis model of the column under explosion impact is established, which provides theoretical guidance for the study of column damage. The main results of this paper are as follows:

- (1) Based on the dimensional analysis method, the formula of the variation in disturbance in the column with the explosion equivalent and the explosion distance is constructed, and a finite element model of a fixed-size reinforced concrete column is established. A numerical simulation is carried out under different working conditions. The simulation data are summarized. When the size of the reinforced concrete column does not change, the disturbance in the column under explosion loading is linearly related to the reciprocal of the proportional explosion distance. Based on existing papers, the linear relationship is verified.
- (2) By determining the form of the variation curve of the middle disturbance with the explosion equivalent and the explosion distance, it can be found that the disturbance in the column decreases with the increase in the proportional explosion distance, and the greater the explosion equivalent, the greater the slope of the straight line. When the proportional detonation distance Z is greater than $0.0693 \text{ m/kg}^{1/3}$, the linear relationship between the column disturbance and the reciprocal of the proportional detonation distance can be used to predict the column disturbance under partial explosion conditions.

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References

1. Wu, W.Y.; Lu, W. Analysis of explosion damage factors in reinforced concrete columns. *Eng. Blasting* **2021**, *2*, 58–65.
2. Jiao, J.F.; Zhao, G.; Hou, Y.L.; Bai, X.H. Dynamic Response of the Concrete Column in Different Restrained Conditions to Resist Blast Load. *Build. Sci.* **2015**, *31*, 60–64.
3. Wei, X.Y. Study on the Response of Concrete and Brick Masonry Materials and Structures under Explosion Impact Load. Master's Thesis, Xi'an University of Architecture and Technology, Xi'an, China, 2007.
4. Su, L.N.; Dong, Y.N.; Hou, Y.L. Numerical Simulation for Effect of Scaled Distance of Concrete Column under Explosive and Impact Loads. *Coal Technol.* **2013**, *32*, 114–117.
5. Peng, Y.L. Dynamic and damage analysis of reinforced concrete frame column under explosion. *Earthq. Eng. Eng. Vib.* **2012**, *32*, 71–78.
6. Yan, J.B.; Liu, Y.; Li, Z.; Xu, Z.X.; Huang, F.L. Effects of Transverse Reinforcement Ratio and Axial Load on the Blast Response of Steel Reinforced Concrete Columns. *Acta Armamentarii* **2020**, *41*, 35–43.
7. Xiao, L.; Qu, W.Z.; Zhou, Y.G. Damage of Frame Structures Subjected to the Loading of Missile Explosion. *J. Wuhan Univ. Technol.* **2010**, *32*, 34–37.
8. Chen, L.; Hu, Y.; Ren, H.; Xiang, H.; Zhai, C.; Fang, Q. Performances of the RC column under close-in explosion induced by the double-end-initiation explosive cylinder. *Int. J. Impact Eng.* **2019**, *132*, 103326. [[CrossRef](#)]
9. He, Y.; Liao, J.Z. Study on Failure Mode and Dynamic Response of Reinforced Concrete Slab Under Blast Load. *Ind. Saf. Environ. Prot.* **2021**, *47*, 32–35.
10. He, Y.; Liao, J.Z. Dynamic response analysis of reinforced concrete beams under blast loading. *Ind. Constr.* **2023**, *53*, 200–202.
11. Wang, W.; Liu, R.C.; Wu, B.; Zhou, S.; Li, L.; Huang, J.; Wang, X. Experimental study of anti-blast performance of three-edge supported square reinforced concrete slab. *Prot. Eng.* **2017**, *39*, 11–17.
12. Chen, X.H. Numerical Research on Dynamic Response of Reinforced Concrete T-beam Strengthened with CFRP Subjected to Blast Load. Master's Thesis, Xiangtan University, Xiangtan, China, 2019.
13. Cui, Y. Dynamic Response and Damage Assessment of Duplex Hollow CFST Column Subjected to Blast Loading. Ph.D. Thesis, Chang'an University, Xi'an, China, 2013.
14. Li, R. Study on the Dynamic Response and Failure Mechanism of Reinforced Concrete Structures Under Strong Shock Wave. Ph.D. Thesis, Tianjin University, Tianjin, China, 2020.
15. Park, G.K.; Kwak, H.G.; Filippou, F.C. Evaluation of nonlinear behavior and resisting capacity of reinforced concrete columns subjected to blast loads. *Eng. Fail. Anal.* **2018**, *93*, 268–288. [[CrossRef](#)]
16. Morrill, K.B.; Malvar, L.J.; Crawford, J.E.; Attaway, S.W. RC column and slab retrofits to survive blast loads. *Adv. Technol. Struct. Eng.* **2000**, 1–8. [[CrossRef](#)]
17. Bao, Y.T.; Tian, K.H. Analytical model of compressive arch action for one-way RC beams and two-way framed substructures under column removal scenarios. *Eng. Struct.* **2024**, *304*, 117694. [[CrossRef](#)]
18. Vanni, N.; Sandro, C.; Fabrizio, G. Nomograms for the pre-dimensioning of RC beam-column joints according to Eurocode 8. *Structures* **2022**, *39*, 958–973.
19. Ma, F.H.; Gillbert, B.P.; Guan, H.; Lu, X.; Li, Y. Experimental study on the progressive collapse behaviour of RC flat plate substructures subjected to edge-column and edge-interior-column removal scenarios. *Eng. Struct.* **2020**, *209*, 110299. [[CrossRef](#)]
20. Chen, Y.Q. Dynamic Analysis of RC Columns Subjected to Blast Loading. Master's Thesis, North China University of Science and Technology, Tangshan, China, 2011.
21. Zhang, X.Z. Dynamic Response of the Stirrup Encryption Reinforced Concrete Columns under Blast Load. Master's Thesis, Xi'an University of Architecture and Technology, Xi'an, China, 2018.
22. Zhao, Z.X. Research on Dynamic Response of Reinforced Concrete Columns Under Contact Explosion and Progressive Collapse Resistance of Structures. Master's Thesis, National University of Defense Technology, Changsha, China, 2023.
23. Shi, Y.C.; Li, Z.X. Dynamic responses and failure modes of RC columns under blast loading. *J. Build. Struct.* **2008**, *4*, 112–117.
24. Li, G.Q.; Qu, H.Y.; Yang, T.C.; Lu, Y.; Chen, S. Experimental study of concrete-filled steel tubular columns under blast loading. *J. Build. Struct.* **2013**, *34*, 69–76.
25. Yan, F.G.; Chen, W.X. Numerical simulation of dynamic response of RPC-FST columns under blast load. *Prot. Eng.* **2019**, *41*, 1–7.
26. Xiang, Q.; Li, J.C. Study on the anti-explosion performance of reinforced concrete columns under explosion load. *Sichuan Archit.* **2011**, *31*, 184–186+189.
27. Han, Z.B.; Qu, W.J. Explosion resistance of hybrid GFRP-steel reinforced concrete slab. *Aata Mater. Compos. Sin.* **2023**, *40*, 6718–6728.
28. Gao, G.F. *Dimensional Analysis: Theories and Applications*; Science Press: Beijing, China, 2021; Volume 12.
29. Xie, Y. Research on Damage Effect of Frame Building under Explosion Load. Master's Thesis, North University of China, Taiyuan, China, 2024.
30. Li, Z.X.; Shi, Y.C.; Zhou, H.Z.; Tian, L. Propagation Law and Overpressure Load of Blast Wave in Urban Complex Environment. *Eng. Mech.* **2009**, *26*, 179–183.
31. Chen, H.; Lu, H.; Sun, S.Z.; Xiong, Z.M.; Yue, S.L.; Wang, D.Z. Damage law of shallow buried reinforced concrete arch structure under secondary explosion of conventional weapons. *Explos. Shock Waves* **2023**, *43*, 155–167.

32. Li, R.L. *Probability Theory and Mathematical Statistics*. Kunming; Yunnan University Press: Kunming, China, 2020; Volume 8.
33. Wang, W. Study on Damage Effects and Assessments Method of Reinforced Concrete Structural Members under Blast Loading. Ph.D. Thesis, National University of Defense Technology, Changsha, China, 2012.

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