

Article

Mechanical Analysis Model of Asphalt Concrete Random Particles Based on Mix Ratio

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Abstract: The strength and other mechanical properties of asphalt mixtures are directly affected by their size, shape, distribution, and interaction of asphalt concrete particles. Mesoscale performance is important for studying the structure of asphalt mixtures. The random particle model of asphalt concrete is one of the important topics in mesoscale research. On the basis of existing research at home and abroad, based on the asphalt mixture mix ratio, this paper further improves the algorithm of random particle generation, optimizes the algorithm of single-particle generation, ensures that the shape of particles at the time of generation conforms to the engineering practice, and forms a random particle reserve library, in order to improve the efficiency of particle delivery, facilitate the model mesh division, and provide a basis for the smooth progress of mechanical analysis.

Keywords: asphalt mixture; random particles; model; algorithm

1. Introduction

The “Outline of Digital Transportation Development Plan” issued by the Ministry of Transport clearly states that China urgently needs to build a digital collection system and promote the digitalization of the whole elements and cycle of transport infrastructure planning, design, construction, maintenance, and operation management. It is particularly critical to focus on the internal structural attributes of physical entities to establish a digital information model to realize the analysis, regulation, and optimization of the unique properties of materials [1,2]. In the study of the micromechanics of asphalt concrete, the size, shape, distribution, and interaction of particles directly affect the mechanical properties of asphalt mixtures, such as strength. The microstructure of concrete directly affects its macroscopic mechanical properties [3–11]. A large number of domestic and foreign scholars have studied the mechanism of the influence of mixture structure on its performance at the microscale using two main methods: analytical and numerical [12,13]. The analytical method is based on micromechanics methods, including direct methods based on representative volume elements, various approximate models based on the theory of single inclusions, and boundary methods based on variational principles [14–16]. These methods require certain assumptions to be set, which are suitable for situations where the integration number of particle phases is relatively small, and cannot consider the random distribution characteristics of inclusions. The numerical method mainly uses digital image technology or parametric modeling methods to establish a simulated real model of asphalt mixtures, and then uses numerical methods for analysis and calculation [17–20]. The particle volume fraction can be large and randomly distributed, making it more suitable for studying the mechanical properties of asphalt mixtures [21–25]. Parametric modeling randomly generates particles of various shapes by selecting appropriate parameter ranges and, through certain shape judgments, forms a single particle that is close to the actual situation. Based on the idea of equal probability placement, a placement area is randomly generated, and particles are selected for random placement, thus establishing a three-dimensional random particle numerical model for asphalt concrete [26–30]. Although



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the shape and position of particles in the model established by this method may differ from reality, its advantage lies in its ability to simulate three-dimensional structures with good repeatability and strong controllability. Compared with digital image technology, parameterized modeling methods can easily achieve three-dimensional modeling of asphalt mixtures while also controlling the microstructure parameters and composition ratio of particles, which is conducive to the optimization design of asphalt mixture performance.

In summary, in the study of the micromechanics of asphalt concrete, the process of particle placement in the three-dimensional random particle numerical model of asphalt concrete is the core step in generating a random particle model. It is very important to take the proper particle placement method that meets the requirement of the particle shape, which helps to greatly improve the construction efficiency of the three-dimensional random particle numerical model. In this study, the fast algorithm was chosen as the placement method, which met the requirement of the polymer particle generated by the improving particle generation method and contributed to the utilization of the real cases. Based on existing domestic and foreign research, this article further improves the algorithm for generating random particles and optimizes the algorithm for generating individual particles, ensuring that particles do not form concave particles during generation and reducing sharp particles, which improves the efficiency of particle placement, facilitates the meshing of the model, and provides a foundation for the smooth progress of mechanical analysis.

To introduce the construction process clearly, the study was divided into four parts, including the construction of the particles based on the mix ratio, the placement method for the polymer particle, the construction of the three-dimensional model, and the application case of the random model.

2. Production of Single Particles

The generation steps of random particles first involve the generation of individual particles, followed by the formation of a backup option library for particles, and finally, the selection and combination of particles according to requirements.

In practical engineering, the particles of the mixture are irregular polyhedra. In order to match the simulated particle shape with the actual situation, this study uses the simplest triangular rotation transformation to form a convex geometry based on geometric significance, and extends the edge line on this basic unit to form particles that conform to the particle size characteristics of asphalt mixtures.

2.1. Efficiency Improvement of the Improved Take-and-Place Method

Based on geometric conditions and particle characteristics, the radius of the outer circle of the triangle can be determined first, and then the three corners of the triangle can be randomly generated to determine. According to the actual material requirements, the content of needle-shaped particles in asphalt concrete aggregates is relatively low, so acute triangles should be used as the basic unit.

By controlling the size range of the three angles of a triangle, the coordinates of points A, B, and C of the plane triangle can be calculated using the semi-valent relationship between the triangle and the circumscribed circle, as shown in Figure 1.

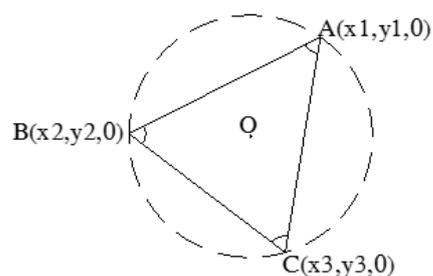


Figure 1. Relationship between simple triangle and circumcircle.

2.2. Transformation Principle of Convex Geometry

According to geometric principles, adding two arbitrary points on the outer side of the triangle plane can form a convex geometry (as shown in Figure 2). In order to avoid generating concave particles during particle generation, the connecting line between the points on both sides of the plane triangle and the intersection point of the triangle plane should be within the triangle. To meet this requirement, first, select the intersection point between the straight line and the plane, and then randomly select points D and E on both sides of the straight line based on the characteristics of the particles. According to relevant experience, when the intersection point between the plane where the triangle is located and the connecting line on both sides is limited to the inner circle of the triangle ABC, the generated particles not only meet the requirements of concavity and convexity but also can approach the particle size.

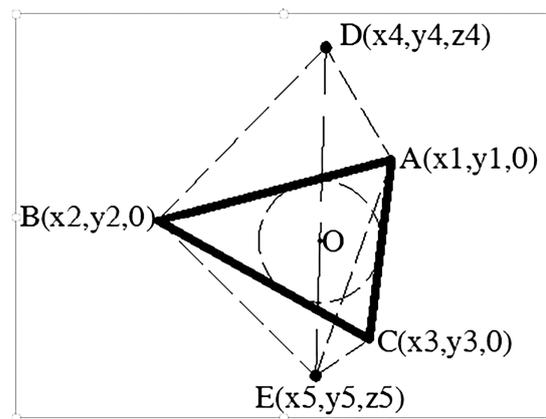


Figure 2. Transformation Principle of Convex Geometry.

When selecting the points on both sides of the triangle ABC, the distance between the points on both sides and the plane should be limited to a certain range; otherwise, needle-like particles may occur and cannot meet the characteristics of the mixture particles. As shown in Figure 3, h is the distance between D or E and plane ABC. After testing, in order to make the generated particles close to the actual particles of the mixture, it is advisable to control h between $0.3 R$ and $1.2 R$ during generation.

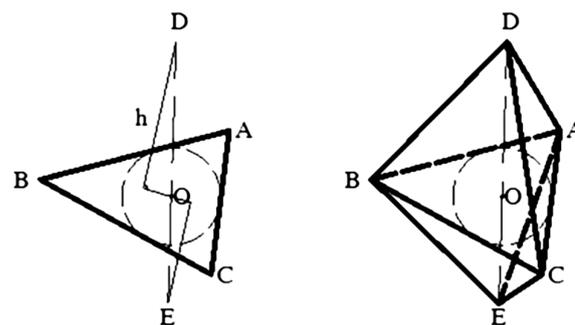


Figure 3. Convex geometry transformation process.

2.3. Formation of Actual Particles

A, B, and C will form a smooth regular plane in the same plane. In order to better match the actual situation of the mixture particles, this study attempts to perform a base point transformation on convex geometry. The plan is divided into two steps to achieve this: the first step is to perform a base point transformation on the xy plane for the convex geometry; the second step is to transform the convex geometry into a base point on the zy or zx plane, as is shown in Figures 4 and 5.

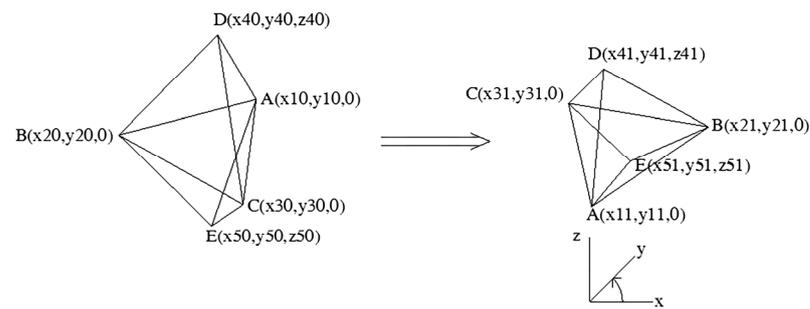


Figure 4. Base point transformation of convex geometry in the xy plane.

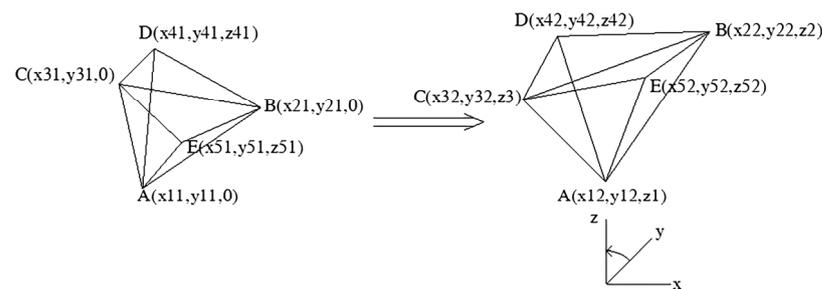


Figure 5. Transformation of convex geometry at yz base point.

Generally, by controlling the cosine value of the triangle, the generated random triangle is an acute triangle. Any polyhedron is derived from the variation of convex geometric particles, which determine the particle size scale and the basic geometric final shape of the particles.

2.4. Expansion of Convex Geometric Particles

To ensure the diversity of particles in the formed mixture, it is necessary to randomly transform the particles. Generally, particles are extended to achieve this. There are generally two ways to implement the extension, extending along an edge of the geometry or extending along a face of the geometry. This study adopts face extension.

For convex geometry, select a point outside the body for expansion. To comply with the particle size of the mixture, the distance between the selected point and the convex geometric plane should be limited. For a certain face of a geometric body, randomly select a point outside the face to expand to generate a tetrahedron, and decide whether to continue expanding based on whether the area of each face of the tetrahedron is greater than the limited area. If $A_i > A_{max}$, then the face will be extended as the base face until all faces meet $A_i < A_{max}$, and then the extension will stop. A_{max} is the predetermined maximum area, which is related to the particle size. At the same time, it is specified that the maximum number of faces for particle expansion is 14. After all the faces that meet the conditions are expanded, if there is still $A_i \geq A_{max}$ among them, it indicates that the selection of base particle parameters is not appropriate, and new base particles should be discarded and regenerated. This cycle continues until a single particle that meets the conditions is generated.

As shown in Figure 6, randomly select the face ADC for expansion. If the ADC area is smaller than A_{max} , select point F for expansion, where the distance between F and ADC must meet the set range. As a result, the particles change from convex geometry to octahedron. Afterwards, determine the area of each octahedron surface. If a certain face is greater than A_{max} , select a point outside its face to expand until all faces meet the requirements $A_i < A_{max}$, and then stop expanding, as shown in Figure 7.

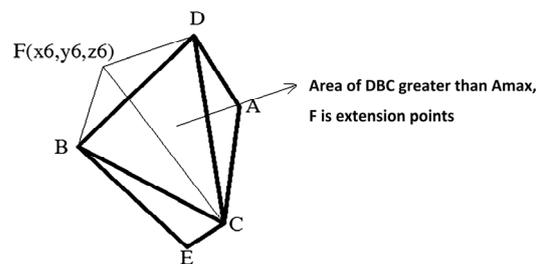


Figure 6. Surface extension of convex geometry.

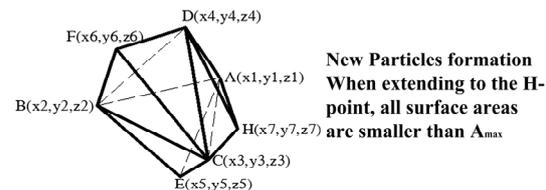


Figure 7. Formation of mixture particles.

2.5. Correction of Elongated and Flaky Particles

The proportion of elongated and flaky particles in actual production is small. At the same time, when these particles perform Boolean operations with asphalt models in modeling, the efficiency of the asphalt model mesh division will become lower, and the number of grids will also increase, which will lead to low computational efficiency. Therefore, further correction should be made for elongated and flaky particles.

(1) Extraction of elongated and flaky particles.

For any particle that has been expanded, the radius of the outer circle that generates its plane triangle is R , the center point of the circle is O , and the vertices that make up the particle are denoted as P_1, P_2, \dots, P_n .

Calculate the distance between any point of the particle and point O . If $d_i > kR$, make a correction to the vertex of the particle. k is the coefficient. After trial analysis, the particle shape obtained when k is between 1.8 and 2.2 is more in line with the actual situation. In this study, $k = 2.0$ is taken.

(2) Method for correcting elongated and flaky particles.

As shown in Figure 8, assuming point P is not satisfied, it is necessary to correct the particles. The specific operation is to cut off a portion of the polyhedron with P as its vertex. The points connected to point P are A, B, C, D , and E . Calculate the three shortest points on the line segment connecting point P , and set them as A, B , and C . Select the midpoints of line segments $P-A, P-B$, and $P-C$ as A_1, B_1 , and C_1 , and randomly select three points A_2, B_2 , and C_2 between line segments $P-A_1, P-B_1$, and $P-C_1$. Cut off the polyhedron using plane $A_2-B_2-C_2$. Obtain new vertices P_1, P_2, P_3, P_4, P_5 , and modify the triangle planes $P-B-A, P-A-C, P-C-D, P-D-E$, and PEB to $P_1-B-A-P_2, P_2-A-C-P_3, P_3-C-D-P_4, P_4-D-E-P_5$, and $P_5-E-B-P_1$. The needle-like particles can be corrected, which is beneficial for the mesh division after modeling.

2.6. Storing Single-Particle Data

After the particles are corrected by elongated and flaky particles, the generation of particles is completed, and then the coordinates and shape data of the particles need to be stored. Each point, surface, and volume in the finite element method have corresponding coordinates. For asphalt concrete specimens with a large number of particles, it is necessary to establish corresponding data for each particle separately. Therefore, for each particle, it is not only necessary to store the coordinates of its vertices, but also the number of the points that form each face, and the number of the faces that form each body.

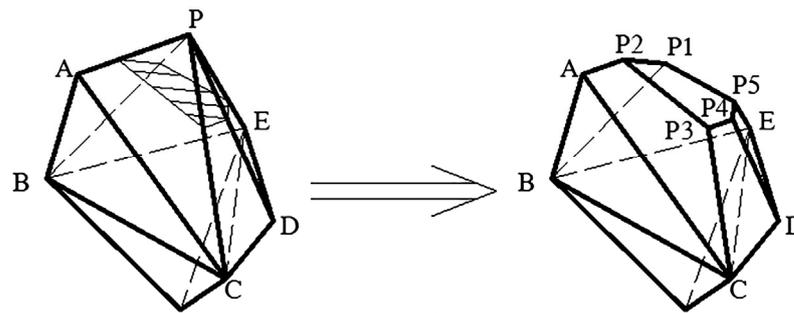


Figure 8. Correction of elongated and flaky particles.

3. Granular Backup Option Library

To simulate asphalt mixtures, it is necessary to determine the content of particles within each particle size range based on the grading curve. If the numerical modeling of asphalt mixtures cannot simulate their true grading curves, further in-depth research on the microstructure behavior of asphalt mixtures will not be possible. To generate a good database of particle backup options that consider actual grading, it is necessary to complete the following tasks: (1) The particle size range in the particle backup option library can provide sufficient particle sizes within the grading curve range; (2) The particle backup option library should have an efficient access method that can quickly and accurately locate the designated particles during the placement process.

After giving the particle grading curve and the percentage of particle volume to the total volume, the total volume of particles within each range can be calculated as a percentage of the total volume. Then, particles of the specified size can be randomly generated based on the particle size range, and the volume of each particle can be calculated for stacking. When the total volume of particles within this range exceeds the specified percentage of the total volume, the generation of particles within that range will be stopped, and the generation of particles within the next range will proceed. The specific process is shown in Figure 9 (Using AC13 as an example).

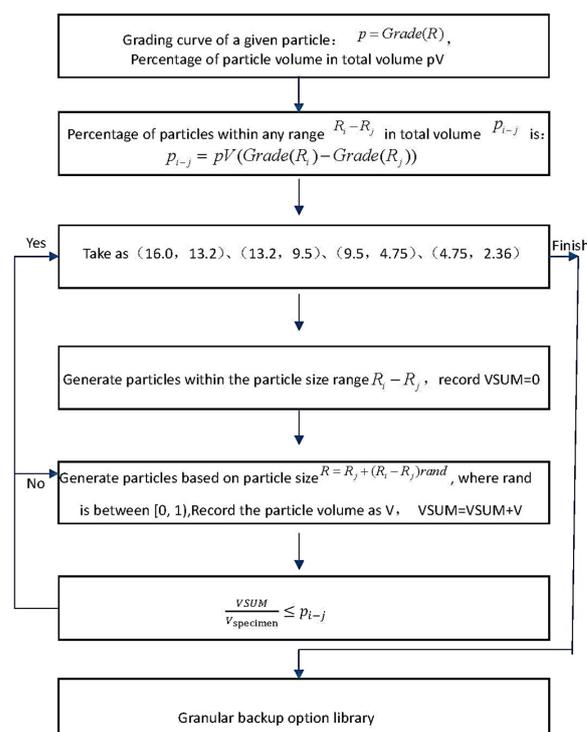


Figure 9. Block diagram of particle backup option library formation.

4. Particle Placement

4.1. Placement Order

Particle placement is the most critical step in the generation of random particle specimens, which not only involves the utilization of the backup option library for generating particles, but also affects the efficiency of specimen generation.

During the process of particle placement, due to the random placement, the sample space becomes smaller. Therefore, it is generally placed according to the particle size from large to small, in order to better ensure the utilization of the backup option library for generating particles, as is shown in Figure 10.

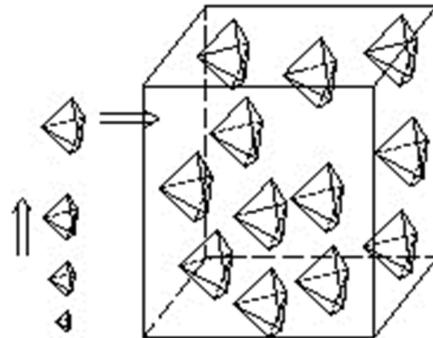


Figure 10. Schematic diagram of placement sequence.

4.2. Conflict Rules

During the placement process, particles cannot directly collide with each other (spatial polyhedra appear to intersect). The key to successful placement is to determine whether particles intersect during the placement process, as there are numerous particles and frequent determination of particle conflicts is required during the placement process. It can be said that the process of releasing particles is the process of determining whether the particles are in conflict with each other. Thus, selecting an effective conflict rule becomes a key element in the efficiency of the entire placement process.

This article is based on the fast algorithm proposed by Ren Shijun [31,32]. The main process is as follows:

- (1) Four parameters are used to represent particles in three-dimensional space, with the following format:

$$ax + by + cz = d \quad (1)$$

So, any point in space can be represented as:

$$ax + by + cz \leq d \quad (2)$$

or:

$$ax + by + cz > d \quad (3)$$

- (2) Any plane in a particle can be represented by three points: $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, and $P_3(x_3, y_3, z_3)$. The four parameters a , b , c , and d in these three-point plane equations can be represented by coordinate values P_1 , P_2 , and P_3 :

$$\begin{cases} a = y_1z_2 - y_1z_3 - y_2z_1 + y_2z_3 + y_3z_1 - y_3z_2 \\ b = -x_1z_2 + x_1z_3 + x_2z_1 - x_2z_3 - x_3z_1 + x_3z_2 \\ c = x_1y_2 - x_1y_3 - x_2y_1 + x_2y_3 + x_3y_1 - x_3y_2 \\ d = -(-x_1y_2z_3 + x_1y_3z_2 + x_2y_1z_3 - x_2y_3z_1 - x_3y_1z_2 + x_3y_2z_1) \end{cases} \quad (4)$$

- (3) After obtaining the parameters a , b , c , and d , it is necessary to write the plane in an inequality format $ax + by + cz < d$, if the points (x, y, z) within the particle satisfy $ax + by + cz > d$. If not satisfied, it will be recorded as $-ax - by - cz > -d$;

- (4) Then, the entire particle polyhedron can be represented as the inequality GL :

$$GL = \begin{cases} a_1x + b_1y + c_1z \leq d_1 \\ a_2x + b_2y + c_2z \leq d_2 \\ \dots\dots \\ a_{n-1}x + b_{n-1}y + c_{n-1}z \leq d_{n-1} \\ a_nx + b_ny + c_nz \leq d_n \end{cases} \quad (5)$$

- (5) Then, any two particles can be written as an inequality GL_i and GL_j , and the two inequalities can be combined to obtain the inequality GL ;
 (6) Determine whether the inequality GL is empty. If it is empty, the two particles do not conflict; otherwise, the two particles intersect.

4.3. Placement Process

As the space available for placement decreases during the placement process, it becomes more difficult to find the placement point of the final particles, resulting in the need for more cycles in the program. During the process of particle placement, it is necessary to determine whether the particles have left the boundary and whether they conflict with the position of the already placed particles. If there is a conflict, it needs to be re-launched. This article selects the following steps for dispensing to achieve fast and accurate dispensing of particles.

- (1) The particles in the generated particle backup option library are sorted in descending order of particle size, and the larger particles are added first when adding them. Set the total initial number of drops to $M = 0$, and the number of single particles drops to DN . The maximum number of times a single particle drops to $MAX1$, and the maximum number of times a total particle drops to $MAX2$.
- (2) Set the particle placement for this time as S_i , radius is R_i , $DN = 0$.
- (3) Randomly generate a delivery point P_i , and determine whether the particle has exited the boundary after delivery. If the delivery point does not meet the requirements, select the delivery point again until the requirements are met.
- (4) Set any particle that has been placed as S_j , and L is the distance P_i from P_j . Record the particle number $R_i + R_j > kL$ that is valid (due to the difference in particle size between a polyhedron and a sphere, the coefficient k is taken as a value less than 1. In order to select all possible and intersecting particles, this article has conducted multiple experiments and selected 0.5), and stored the recorded particle number in the matrix KS .
- (5) Determine S_i if there is a conflict with particles in KS . Conflict: then, $DN = DN + 1$. If $N > MAX1$, adjust to (8); otherwise, skip to (3). No conflict: This placement was successful, and the placement point P_i is the center point of S_i .
- (6) Number the particles that have been placed in sequence, and record them in matrix S . Record the center point of the particles in matrix P , and record the particle radius in matrix R .
- (7) $M = M + DN$, if $M \geq MAX2$, end placement, return.
- (8) Enter the next particle placement, and the results is shown in Figure 11.

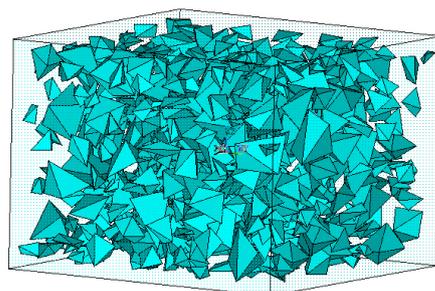


Figure 11. Placement results.

5. Model Building

The random particle model of asphalt concrete is a simplified model for pavement asphalt concrete specimens at a microscopic level. This model simulates the random distribution of particles in asphalt in reality, which can be closer to the actual situation of asphalt concrete and can also accurately obtain theoretical data.

This article studies a mesoscopic random particle model. There are hundreds of thousands of units. If modeled according to the actual size, the number of model units will be very large, so it is necessary to reduce the simulation of the actual model.

The random particle model of asphalt concrete in this article has a shape of $80\text{ mm} \times 80\text{ mm} \times 60\text{ mm}$ rectangular body. Before randomly generating particles, it is necessary to specify the origin of the coordinate system to facilitate modeling and post-processing analysis. The model established in this article is a rectangular asphalt concrete specimen, with the middle plane of the specimen as the xoy plane. The established coordinate system is shown in Figure 12.

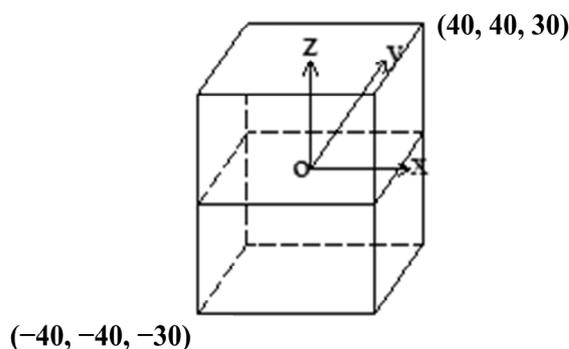


Figure 12. Coordinate system and rectangular body.

Limestone particles were selected, with an elastic modulus of 40 GPa and a Poisson's ratio of 0.2. The particle models used in this article are all convex particles and all are polygonal particles. SLOID45 element is used to simulate particles (Figure 13).

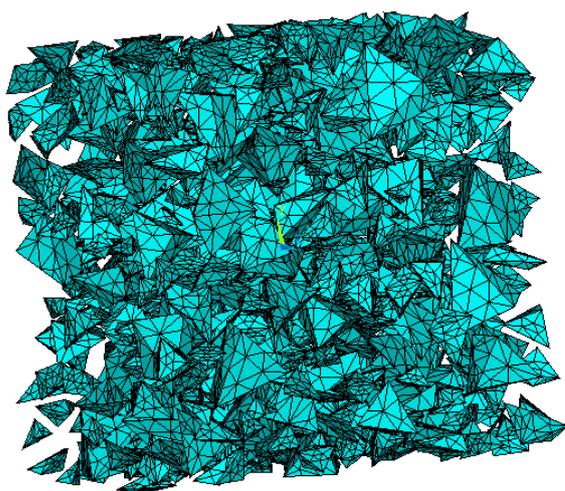


Figure 13. Particle division.

Asphalt sand adopts the viscoelastic Burgers model. The various parameters of the asphalt sand used in this article are shown in Table 1.

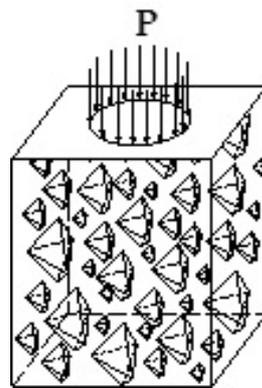
In order to approach the actual situation, the uniformly distributed load on the surface of the specimen model is equivalent to the actual tire pressure, and the area of action of the uniformly distributed load is reduced by the corresponding multiple of the model.

Table 1. Parameters of the asphalt sand.

| T (°) | g_1 | g_2 | τ_1 |
|-------|--------|--------|----------|
| 4.4 | 0.5899 | 0.4101 | 89.9143 |
| 64 | 0.0094 | 0.9906 | 14.1169 |

The tire pressure P is 0.707 MPa, so the uniformly distributed load is taken as 0.707 MPa. The load is a uniformly distributed circular load with a radius of 10 mm, with a vertical downward direction.

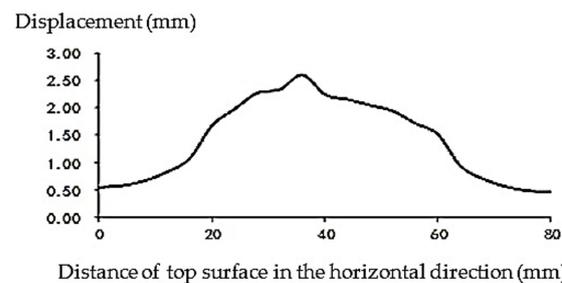
Constrain the normal displacement on all sides of the specimen, and also constrain the normal displacement on the bottom surface of the specimen. The top surface under load is a free surface, the schematic diagram is shown in Figure 14.

**Figure 14.** Loading method of specimen model.

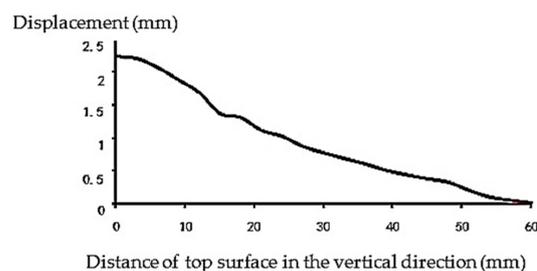
6. Numerical Simulation

6.1. Displacement

The change in displacement of the top surface in the horizontal direction of a straight line along the x -axis direction is shown in Figure 15.

**Figure 15.** Displacement of x -axis direction of the top surface.

The displacement change in the vertical direction of the center of the specimen is shown in Figure 16.

**Figure 16.** Displacement of Z -axis direction at the center of the specimen.

The displacement diagram of Figures 15 and 16 indicates that there is a certain displacement change in the asphalt pavement displacement considering aggregates in a relatively homogeneous model, but the overall trend is consistent.

6.2. Stress

Study the variation of shear stress in straight lines ($x = -40, y = 0, z = 0$) – ($x = 40, y = 0, z = 0$). The variation diagram of shear stress S_{XZ} , S_{YZ} , and S_{XY} with position is shown in Figures 17–19.

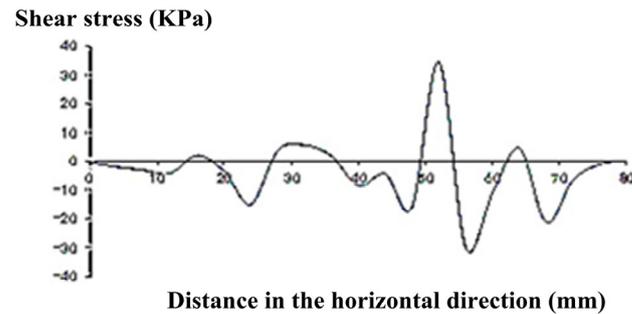


Figure 17. Variation of shear stress S_{XZ} .

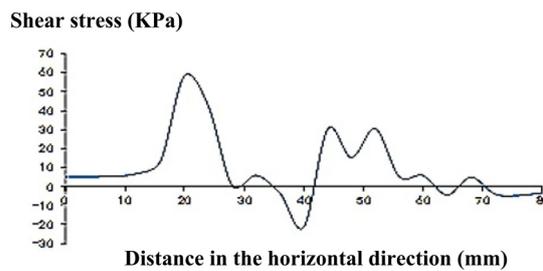


Figure 18. Variation of shear stress S_{YZ} .

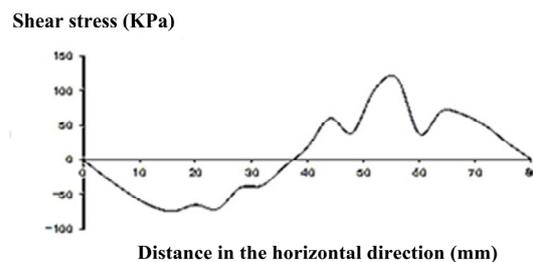


Figure 19. Variation of shear stress S_{XY} .

Study the variation of tensile stress in straight lines ($x = -40, y = 0, z = -30$) – ($x = 40, y = 0, z = -30$). The variation diagram of tensile stress with position is shown in Figure 19.

The stress diagram of Figures 17–19 indicates that there is a significant correlation between the stress of asphalt concrete pavement and the changes in particle arrangement when considering particle stress. This provides a good mechanical analysis tool for the microscopic study of particles and their connections, laying the foundation for future research.

7. Conclusions

Based on the asphalt mixture ratio, this paper further improves the algorithm for generating random particles, optimizes the algorithm for generating individual particles, ensures that the shape of particles during generation conforms to engineering practice, forms a random particle backup library, proposes a particle placement method, and forms a random aggregate model for asphalt concrete. The model is used for numerical analysis.

The result shows that when considering particle forces on asphalt concrete pavement, there is a significant correlation between its stress and changes in particle arrangement. This study provides a good mechanical analysis tool for the microscopic study of particles and their connections, laying the foundation for future research.

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Data Availability Statement: Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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