


## Article

# Social Pressure in Networks Induces Public Good Provision

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**Abstract:** I develop a dynamic model with forward looking agents, and show that social pressure is effective in generating provision in a public good game: after a small group of agents start contributing to the public good, other agents decide to contribute as well due to a fear of being punished, and this generates contagion in the network. In contrast to earlier models in the literature, contagion happens fast, as part of the best response of fully rational individuals. The network topology has implications for whether contagion starts and the extent to which it spreads. I find conditions under which an agent decides to be the first to contribute in order to generate contagion in the network, as well as conditions for contribution due to a self-fulfilling fear of social pressure.

**Keywords:** social pressure; public goods; contagion

## 1. Introduction

Why do public good provision and collective action take place? Given that collective action and public goods are by definition non-excludable, most individuals have no incentive to participate [1,2]. One possibility is that people participate out of a fear of social sanctions. For example, large numbers of young British people enlisted in WWI out of social pressure ([3], pp. 115–116). There is now a wealth of empirical evidence pointing out the role of social pressure in the provision of a range of public goods, such as voting [4–7], environmentally-friendly behavior [8–11], costly activism [12], charity donation [13], and so forth.

I propose a model that is explicitly based on social pressure: individuals are embedded in a network, and they receive disutility when their friends contribute to the public good but they do not. Agents face a decision of whether to contribute to a public good: the benefit  $b$  is public, but the cost  $c$  is private and larger than the benefit ( $c > b$ ). At the beginning of the game, nobody is contributing to the public good; the game is dynamic, and every period the game ends with constant probability. If it continues, an agent is selected randomly to revise her strategy. This simple model has interesting implications. Whenever a group  $Y$  of agents contributes to the public good, there is an immediate contagion in the social network—initially, friends of those in  $Y$  might find optimal to contribute, as they do not want to suffer social pressure; however, they cannot revise their strategy until they are randomly chosen to do so. Friends of friends of those in  $Y$  know that friends of those in  $Y$  will contribute as soon as they are able to revise their strategy. They measure their potential expected disutility from social pressure, and they compare it with their certain cost of contributing to the public good: if the former is greater than the latter, they also decide to contribute whenever they get to revise their strategy. As more and more agents perform the same reasoning, a contagion in best responses is generated, by which most individuals in the network end up contributing. The earlier literature explained contagion over time [14,15] focusing on transitions between different equilibria: every individual best-responds to everybody else in society, and behavior changes from generation to generation, due to random shocks that shift behavior between different basins of attraction. In contrast, in this paper behavior change happens *as part of the equilibrium*—as agents contribute, they increase the incentives for other agents to contribute due to social



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pressure, generating a snowball effect. Crucially, unlike Kandori et al. [14], Young [15] and Morris [16], in which behavior change happens as agents change their selected action, in this paper contagion happens instantly as part of the best response of the individuals in the network<sup>1</sup>. Therefore, rather than analyzing how different conventions evolve over generations, I focus on how behavior can change almost instantly, due to contagion in the network induced by social pressure.

The fact that social pressure can induce public good provision has implications for a long-standing literature that has studied revolts and other forms of political collective action, by assuming that individuals derive a *private benefit* from the success of the revolt [17–21]. One of the theoretical contributions of this paper is to provide a mechanism that justifies participation in revolts and other forms of collective action, even in the absence of such private benefit, as long as there is social pressure to participate. This paper therefore reinforces the results of the aforementioned literature, by providing a micro-foundation of why individuals would be willing to participate in revolts or other forms of political activism in the first place<sup>2</sup>.

Leaders are important for collective action. For example, Rosa Parks is regarded as a spearhead of the civil rights movement in the United States: in 1955, she refused to give up her seat on a bus in protest at a racist law that segregated seats in public transportation, an act that sparked a movement that eventually led to major advances in civil rights in the United States [22]. In terms of the model, I call the first individual to contribute the *leader*, and I show that under some conditions agents will decide to become leaders, generating a large contagion and hence a large contribution in the network as an outcome. This happens when the contagion generated by an agent  $i$  is so large that the expected benefit from inducing others to contribute is larger than the private cost of contributing: the contribution to the public good by the leader herself is negligible but, because she manages to spread the contagion to a large fraction of the population, she has incentives to contribute even when nobody has contributed before. It can also happen that when several agents are afraid of social sanctions by others, those agents contribute (thereby punishing those who do not contribute); therefore the fear of social sanctions becomes self-fulfilling. The question of leadership is rich and complex, and there is a large literature that has addressed it in the context of teams (see Reference [23], for a survey). Precisely because of this complexity, there are only a handful of papers that have considered leadership in a larger setting, where a single individual can tip the equilibrium of the whole society. In a paper of technology/behavior diffusion, Morris [16] takes the innovators as exogenous. As in the model presented in this paper, Ellison [24] considers the case in which a single rational player can generate contagion to a whole population of myopic agents, and Corsetti et al. [25] also consider the importance of a single player; however, both of these papers take leaders as exogenous. Reference [26] considers a case where an individual can change a social norm, and that social norm will stick to at least some future generations; leaders in their model are endogenous, but only a special class of individuals can become leaders. A contribution of the present paper is to analyze how and when a “regular” individual endogenously becomes the leader of a large group.

One of the most intriguing questions in Economics is the evolution of cooperation in humans, that is, how is it that provision of public goods and other cooperative behaviors is so widespread in human populations, despite the obvious possibilities of exploitation by selfish individuals [27]. Recent studies have showed that networks might play a crucial role in answering this question—for example, the evolution of cooperation can be successful if the ratio of benefit to cost  $b/c$  is larger than the average number of neighbors in the network [28]. The present paper offers a complementary approach that emphasizes

<sup>1</sup> Note that contagion will happen instantly, in the sense that most agents will choose to contribute whenever they are given the option to revise the choice. However, they will have to wait for that option to revise their strategy (that happens stochastically), before they can change their action.

<sup>2</sup> Reference [12] argues that the benefit  $b$  comes from the mobilization of personal ties in preexisting social networks. By explicitly separating the public benefit  $b$  from the utility of mobilizing (or keeping) personal ties, the present paper makes clear when is it that social pressure is effective in generating collective action.

the role of forward looking behavior of agents, and that therefore can make cooperation even more likely than in the case of myopic individuals. This paper is connected to several other important literatures. Miguel and Gugerty [29] consider public good provision with social sanctions that are a function of the share of contributors in the whole population, unlike in the present model in which social sanctions happen locally in the network<sup>3</sup>. Karlan et al. [32] and Jackson et al. [33] emphasize the idea that links in the social network are valuable, because they provide a “collateral” in social interactions. The present paper is similar in spirit to the idea that a person can use her links in the social network as valuable assets, and applies this idea to public good provision<sup>4</sup>. Calvó-Armengol and Jackson [37], Ali and Miller [38,39] consider cases in which individuals only observe their neighbors, and the punishment behavior is endogenous; these assumptions are the opposite to the ones in the present model, making our results complementary, as they are extreme cases of what is likely to happen in real-world networks. Bramoullé et al. [40] provide a general characterization of local-interaction games, in which an agent’s payoff depends linearly on the actions taken by her neighbors, and Allouch [41] analyzes a similar framework in which individuals’ payoffs depend non-linearly on the public good contributed by their neighbors. These papers are essentially static and consider that the public good is limited to the agents’ neighborhood; in contrast, I focus on dynamic best-responses generated by local social pressure of a global public good. Galeotti and Goyal [42] consider the question of how to influence the agents in the network, and Siegel [43] analyzes how the network topology can affect collective action. Finally, this paper is related to the seminal literature in social dynamics [14–16] and the theoretical literature on technology adoption, where it is usually found that a small group of connected individuals can foster technology adoption in the network [44–48]<sup>5</sup>. I consider fully rational and forward looking agents, which makes contagion fast, and generated by leaders who do so in a calculated manner. A remarkable branch of this literature [50–53], considers agents who are forward looking and can only revise their strategy as a Poisson process. However, they consider a setting in which the action of any single agent does not affect the payoff for the rest of the population, and because of that, there is no contagion or leadership considerations.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 analyzes the contagion that takes place in the network, once agents have started contributing. Section 4 analyzes under which conditions agents become leaders by contributing when nobody has contributed before, as well as when contribution happens due to a self-fulfilling fear of social pressure. Section 5 concludes. The Appendix A provides proofs and details.

## 2. The Model

There is a finite number  $n$  of agents in the society, represented by set  $N$ . The structure of society is given by a friendship network  $g \in \{0, 1\}^{N \times N}$ , where  $g_{ij} = 1$  if agents  $i$  and  $j$  are connected, and  $g_{ij} = 0$  otherwise<sup>6</sup>. The network is undirected ( $g_{ij} = g_{ji}$ ), that is, friendship is reciprocal. Moreover, the network is *connected*, that is, any agent  $i \in N$  is a friend of at

<sup>3</sup> In support of this intuition, References [30,31] have documented the importance of social pressure and social interactions in the context of savings associations.

<sup>4</sup> The literature on public good provision has tried to analyze how different characteristics of the population (such as ethnic, racial and socioeconomic heterogeneity), affect the level of public good provided in equilibrium [34–36]. The results of this literature show that heterogeneity in the mentioned characteristics is associated with lower public good provision. Even though the present model does not include heterogeneity along those dimensions, I believe that it points to a causal mechanism: public good provision would indeed be lower if heterogeneity across ethnicity, socioeconomic status, and so forth generates social norms with less sanctions, or makes it more likely that the network is fragmented in cliques that do not interact with each other (see Example 3 below). Each of those cases would make social pressure not operational, and hence public good provision to decline.

<sup>5</sup> The model presented in this paper is especially related to Morris [16] in how contagion spreads through a network as a function of the network characteristics (see also Reference [49]).

<sup>6</sup> Throughout the paper I use the terms *friends* instead of the more common terminology *neighbors* for individuals who are connected, to make the discussions easier to read.

least another agent in  $N$ . Let  $N_i$  denote the set of agent  $i$ 's friends:  $N_i = \{j \in N : g_{ij} = 1\}$ . Each agent faces a binary choice: to contribute or not to contribute to the public good. Time is discrete,  $t = 1, 2, \dots$ , and each period only one agent (selected at random) can revise her strategy. Therefore, every agent must play the same strategy in every period until she has the opportunity to revise (for example, individuals can only enlist in the military or participate in a protest whenever there is an opportunity to do so). The game can be summarized every period  $t$  by a state  $S^t \in \{0, 1\}^N$ , where  $s_i^t$  denotes the action last chosen by agent  $i$ , and  $s_i^t = 1$  means the agent contributed. Initially, that is, at  $t = 0$ , we have  $S^0 = \{0\}^N$ , that is, every agent starts the game not contributing. At the beginning of each period, the game continues with exogenous probability  $q$ , therefore, the game ends with probability  $1 - q$ , in which case payoffs are realized. If the game continues, an agent  $i \in N$  is selected at random to play, from a i.i.d. uniform distribution on the set of agents  $N$ . Agent  $i$  at period  $t$  chooses an action from  $A_i(S^t)$ , where

$$A_i(S^t) = \begin{cases} \{0, 1\} & \text{if } s_i^t = 0, \\ \{1\} & \text{if } s_i^t = 1. \end{cases}$$

This means that once an agent has contributed, she must stick to that action for the rest of the game. Intuitively, once an individual has enlisted in the military (or participated in a violent protest) it is extremely difficult or impossible to undo such action (this assumption is also typical of the literature on innovation diffusion [48])<sup>7</sup>. Given this structure, the game is such that agents play one at a time, selected at random with replacement from the set  $N$ , until the game ends. When the game ends and the state is  $S^T$  for some final period  $t = T$  (that is itself random), payoffs are realized, according to the following utility function, where  $\mathcal{I}_i$  indicates whether the agent had a chance to play at least once prior to the game's end<sup>8</sup>:

$$u_i(S^T) = b \left( \sum_{k \in N} s_k^T - \psi(1 - s_i^T) \mathcal{I}_i \sum_{j \in N_i} s_j^T \right) - cs_i^T. \quad (1)$$

Agent  $i$  derives utility  $b$  from each agent who contributed and pays the private cost  $c$  if she herself contributed<sup>9</sup>. Moreover, agent  $i$  suffers **social pressure** whenever she does not contribute to the public good: in that case she incurs a disutility  $b\psi$  for each friend who contributed. Therefore, the total social punishment is proportional to the importance of the public good  $b$ , the social pressure parameter  $\psi$ , and the number of friends who contributed<sup>10</sup>. In the example of war,  $b$  would represent the marginal benefit of one more soldier (assumed to be constant) in the defense of the country,  $c$  is the cost the individual faces (the likelihood of being injured or killed, foregone wages, etc.) and  $\psi$  is a scaling parameter that connects the benefit  $b$  with the social punishment for not participating. Note that this punishment is unidirectional: those who participated costlessly punish those who did not participate, as is the case in situations in which there are social norms of "honor" and "shame" for participating or not in a certain activity [58]. With no social

<sup>7</sup> In terms of the model, this prevents the usual punishment strategies in repeated games to play a role. For example, players cannot play a grim trigger strategy, since they cannot undo their contributions. Note that the same could be accomplished by introducing a cost for reversing the participation decision. As long as the cost would be large enough, then the model would be equivalent to the one presented here. I have chosen not to include such "reversing costs", in order to keep the model tractable.

<sup>8</sup> A referee pointed out that those who never had a chance to contribute would likely not suffer social pressure in a realistic scenario, hence I assume that the social punishment only happens when agent  $i$  had a chance to play. Note that this assumption is without loss of generality.

<sup>9</sup> I am assuming that there is a constant marginal benefit  $b$  for the public good. It could be argued that for revolutions and other regime-change games, the marginal contribution is not constant, but rather exhibits a sharp jump at a given threshold (I thank a referee for pointing this out). Since the focus of this paper is on contagion and leadership, I maintain the simple assumption of constant marginal benefit, in order to obtain characterizations of those phenomena. Note that all results could be re-written considering an variable marginal benefit  $b(n)$ , at the cost of making the characterizations more cumbersome.

<sup>10</sup> This assumes that agents value relationships "per se", for example because having friends allows an individual to be connected to other agents in the network. This can be partially justified by the literature in psychology and behavioral economics [54–57]. Despite this evidence, it can also be considered as a "reduced form" utility from a more general model, where having friends signals some underlying disposition, like being honest.

pressure ( $\psi = 0$ ), it is easy to see that nobody would ever contribute in a subgame perfect equilibrium (SPE)<sup>11</sup>. To make this a game of public good provision, I assume that the private cost is larger than the benefit an agent enjoys by contributing.

**Assumption 1.**  $c > b$ .

Given the probability  $q$  that the game continues, we can define  $Q$  as the probability that a given agent  $i$  will be able to play before the game ends (for example, having an opportunity to enlist before the war is over). Because the game continues each period with a fixed probability  $q$ , and since the individual that plays each period is chosen uniformly with replacement from the entire population, it follows that  $Q$  does not depend on time  $t$  or on the identity of the player: hence  $Q$  is constant across periods and players. Moreover, I assume that  $Q$  remains constant for any population size  $n$  (if this was not true, the results would depend artificially on the size of the population, as will become clear in Section 3).

**Assumption 2.** The probability that the game continues  $q$ , as a function of  $n$ , is such that  $Q$  is constant.

Therefore, the probability  $Q$  that an individual will be chosen before the game ends can be recursively written as  $Q = q(n) \left( \frac{1}{n} + \frac{n-1}{n} Q \right)$ . Solving for  $q(n)$ , we obtain:

$$q(n) = \frac{n}{n - 1 + \frac{1}{Q}}. \quad (2)$$

### 3. Contagion

In this section, I analyze how an agent's contribution can affect how others behave in the network, and generate a "wave of contagion" that spreads contribution by best-response dynamics to a large fraction of the network. A very similar effect was analyzed in Reference [16], in the context of myopic agents. The conditions I find are such that even when agents are forward looking, they have as a dominant strategy to contribute. I will use the concept of Subgame Perfect Equilibrium (SPE):  $\sigma$  is a SPE if it is a Nash equilibrium of every subgame; in other words, if whenever an agent  $i$  is chosen to play, she best responds to  $\sigma_{-i}$  [59].

Because agents can only play when they are chosen to do so, we must distinguish between those who have already contributed, and those who would like to do so if chosen to play (but have not contributed yet).

**Definition 1.** An agent is **contributing** if she has chosen to contribute. An agent is **predisposed** if she has not chosen to contribute yet, but will do so the next time she is chosen to revise her strategy.

**Definition 2.** I define the **social pressure threshold**  $\alpha^*$  as the minimal  $\alpha$  that makes it dominant to contribute when an agent has at least  $\alpha$  contributing friends, that is,  $c/b < 1 + \psi\alpha$ .

Consider the case when a set of agents  $Y$  have contributed to the public good, and consider an agent  $j$  who has more friends in  $Y$  than the social pressure threshold  $\alpha^*$ . That means that  $j$  will contribute whenever she can, that is, she becomes predisposed. But note that there might be many other agents like  $j$ , and all of these agents are now predisposed, and will contribute whenever they can. Let  $\tilde{Y}$  be the set of such predisposed agents.

<sup>11</sup> Indeed, suppose that  $n - 1$  agents have contributed, and the last agent has to decide whether to contribute. Because all other agents must contribute in the future (as they cannot change their action to 0 anymore), there is no reputation loss for not contributing. Because  $c > b$ , she will choose not to. But that means that previous actions of other agents cannot change what the last agent will do, and so the second-to-last agent does not contribute either. By induction, no agent will ever contribute.



Things become more interesting when we look at an agent  $i$  that does not have friends in  $Y$  but does have friends in  $\bar{Y}$ . Agent  $i$  is suffering a type of “indirect” social pressure: while none of her friends are contributing yet, they are predisposed and will be contributing in the future. Therefore, agent  $i$  might want to contribute to avoid social pressure preemptively, and therefore might become predisposed as well. Whether agent  $i$  becomes predisposed depends on the cost-benefit analysis, in particular on the social punishment and the cost of contributing with respect to the benefit, and on the likelihood that her friends will end up contributing before the game ends.

This is the basis of contagion in this paper: agents who have predisposed friends can become predisposed themselves, furthering the contagion (of predisposed agents) in the network. As more and more agents become predisposed, it is more likely that the next agent chosen to play will be predisposed, and hence will contribute, what increases contribution in the network.

**Proposition 1.** *If agent  $i$  has  $y$  contributing friends, and  $m$  predisposed friends, a sufficient condition for her to contribute is:*

$$\frac{c}{b} < 1 + \psi \sum_{x=0}^m P(x|m)(y+x), \quad (3)$$

where  $P(x|m)$  is the probability that  $x$  out of  $m$  predisposed friends contribute before the game ends and before agent  $i$  can play again, and can be computed recursively by

$$P(x|m) = \frac{m}{n(q-1) + m + 1} P(x-1|m-1), \quad \text{and} \quad P(0|m) = \frac{1-q}{1 - q^{\frac{n-m-1}{n}}}.$$

The intuition for Proposition 1 is that the cost-to-benefit ratio for contributing (which is higher than 1 by Assumption 1) cannot be too large in comparison with the expected social punishment. If the condition in Equation (3) holds, the best response for the agent is to contribute, and so she becomes predisposed. One interesting thing to note from Proposition 1 is that the agent can decide to contribute, even if she only has predisposed friends (but not necessarily contributing ones). There exists a minimal number  $m^*$  of predisposed friends that makes Equation (3) hold (even in the absence of contributing friends), that is, such that for all  $m \geq m^*$  agents with at least  $m$  predisposed friends have as a unique best response to contribute when chosen to revise their strategy, therefore becoming themselves also predisposed.

**Definition 3.** *The contagion threshold  $m^*$  is the minimal number  $m$  of predisposed friends, that satisfies the sufficient conditions in Equation (3) for an agent to contribute, even when none of her friends are contributing.*

Note that  $m^*$  is decreasing in the public good benefit  $b$ , the social pressure parameter  $\psi$  and the probability that an individual will play before the game ends  $Q$ ; and increasing in the private cost  $c$ . Hence, greater values of  $b$ ,  $\psi$  or  $Q$ , or lower values of  $c$  make contagion more likely. These comparative statics of  $m^*$  with respect to the parameters of the model are intuitive: higher benefit, lower cost, or higher social pressure all induce more contagion. The fact that a higher  $Q$  (higher probability of revising before the game ends) induces more contagion is also clear: not only does this generate more possibilities for agents to contribute, but those possibilities are in turn anticipated by others, and that will make them also more likely to contribute.

I turn now to study how contagion spreads through the network. I use the notation from Reference [16]. Let  $X$  be a set of agents: I define operator  $\Pi_m^0(X) := X$ , and for  $k \geq 1$ , I define  $\Pi_m^k$  recursively:

$$\Pi_m^k(X) = \Pi_m^{k-1}(X) \cup \{i \in N : |N_i \cap \Pi_m^{k-1}(X)| \geq m\}.$$

It is clear that this is an increasing sequence of sets, and I define  $\Pi_m^\infty(X)$  as the limit of that sequence. We can see then what happens when agents are contributing in the network and the contagion threshold is  $m^*$ . Let  $Y$  be the set of all agents who have contributed. Then, all agents who have at least  $\alpha^*$  friends in  $Y$  become predisposed, since  $\alpha^*$  is the social pressure threshold. Let  $\tilde{Y}$  be the set of agents who are either in  $Y$  or have become predisposed because of the actions of agents in  $Y$ . But then, agents who have  $m^*$  friends in  $\tilde{Y}$  also become predisposed, so the contagion spreads to the set  $\Pi_{m^*}^1(\tilde{Y})$ . Proceeding recursively, the contagion spreads through the network and in the end all agents in  $N_{m^*}^\infty(\tilde{Y})$  are predisposed. This contagion happens “instantly”, in the sense that it is not necessary that more agents contribute, but rather the contagion happens by an expansion of best responses that make agents predisposed, even before the next agent has a chance to play, so when next period starts any agent in  $N_{m^*}^\infty(\tilde{Y})$  will contribute if selected to play. Hence we have the following result.

**Proposition 2.** *In any SPE, when agents in  $X$  become predisposed, all agents in  $\Pi_{m^*}^\infty(X)$  become predisposed.*

In particular, if there is a set of contributing agents  $Y$ , and  $\tilde{Y}$  is the set that also includes predisposed agents, then contagion will expand to set  $\Pi_{m^*}^\infty(\tilde{Y})$ . We have seen how a contagion of best responses takes place in the network. This phenomenon is similar to the contagion in References [16,60], and many other models of contagion, with two remarkable differences. Firstly, all agents are forward looking, and contagion takes place in anticipation to the actions of others. This is a contribution with respect to the previous literature, where the individuals are either not forward looking (or only some individuals are, as in Reference [24]), or they are unable to initiate a contagion. Secondly, contagion happens in the same period that agent  $i$  contributes: because players are forward looking, contagion is instantaneous. This is in contrast with the literature of social dynamics, where contagion is supposed to happen in the long run or as the limit of some updating process<sup>12</sup>. Interestingly, both aspects of the present model are related. It is precisely because agents are rational and forward looking that contagion spreads so fast: everyone anticipates that others will also contribute.

Note that despite the potential for multiple equilibria, agents who are affected by contagion have as a dominant strategy to contribute, irrespectively of what non-predisposed agents do. This comes at the expense of obtaining a lower bound on the actual contagion on the network (the conditions I required here are stronger than necessary for a contagion to happen, but if they do happen, we can unambiguously claim that contagion takes place). As argued in the Introduction, I am interested in conditions that would generate collective action, hence the focus on sufficient conditions. I turn now to analyze how (and if) contagion happens in a few interesting networks.

**Example 1 (Regular lattice).** *Consider that agents belong to  $\mathbb{Z}^d$ , where  $\mathbb{Z}$  is the set of integers, and  $d$  is the dimension of the lattice<sup>13</sup>. Agents are friends if they are next to each other, i.e.,  $g_{ij} = 1$  if  $\sum_{k=1}^d |x_i^k - x_j^k| = 1$ , where  $x_i^k$  is the  $k$ -th coordinate for agent  $i$ . In such a regular lattice, contagion does not propagate for  $m^* > 1$ . The reason is that friends of  $i$  are not friends themselves, that is, the clustering coefficient is 0<sup>14</sup>. Therefore, even if the friends of  $i$  become predisposed, their friends will not become predisposed for  $m^* > 1$ .*

<sup>12</sup> See References [61,62], for a discussion about the speed of contagion.

<sup>13</sup> While in this example and the next the networks are infinite, that is just for simplicity. The examples can as easily be understood to be in the set  $\mathbb{Z}_M^d$ , where  $\mathbb{Z}_M$  is the set of integers with absolute value at most  $M$ .

<sup>14</sup> The clustering coefficient is defined as the number of triads of agents who are linked to each other, divided by the possible number of triads. Because no three agents are all linked to each other in a regular lattice, the coefficient is 0.

**Example 2** (“Chess lattice”). Agents belong to  $\mathbb{Z}^2$ , and agents are friends if they are either next to each other, or one step diagonally (as the king moves in chess), that is,  $g_{ij} = 1$  if  $\max_k |x_i^k - x_j^k| = 1$ . In this case, contagion happens for any contagion threshold  $m^* \leq 2$ . The case  $m^* = 1$  is trivial. The case  $m^* = 2$  is derived from the fact that for every friend  $j$  of  $i$ , there exists at least one friend  $k$  who is friend of both  $i$  and  $j$ . Therefore, when both  $i$  and  $j$  become predisposed, so does  $k$ . Since this is true for all agents in the network, contagion ensues. Note that if  $m^* = 3$ , contagion is not guaranteed. For example, if both  $i$  and two of her friends who are at opposing diagonals become predisposed, there is no agent  $k$  who is simultaneously friend of all three predisposed agents<sup>15</sup>.

**Example 3** (Cliquish network). Consider network  $g$ , such that  $g$  is composed by a collection of cliques (a clique  $C$  is a subset of  $g$ , such that if  $i \in C$ , then  $g_{ij} = 1$  for all  $j \in C$ ). In other words, all agents in a clique are friends with everybody else in that clique. Cliques are connected to other cliques via  $l^2$  links, that is,  $l$  agents of clique  $C$  are connected to each of  $l$  agents of clique  $C'$  (and vice versa). It can be showed that contagion will happen in cliquish networks if there are initially  $m^*$  predisposed agents in a given clique  $C$ , whenever  $l \geq m^*$  (of course, assuming that cliques have more than  $m^*$  agents). This is because there are  $l$  agents in clique  $C'$ , each of whom has  $l$  friends in clique  $C$ , and since  $l \geq m^*$ , each of these agents becomes predisposed. But then, agents in clique  $C'$  have at least  $m^*$  predisposed friends, and also become predisposed. This reasoning can be extended to the whole network, assuming that there is a path from clique  $C$  to any other clique.

Until this point, I have considered how contagion spreads when a set of agents  $Y$  contributes. In a sense, contagion exhibits strategic complements: the more likely an agent is to contribute, the more likely others contribute because of the fear of social pressure. However, why would an agent contribute in the first place and become a leader? I answer this question in the next section.

#### 4. Leadership

Hermalin [23] defines a *leader* as someone with voluntary followers, as opposed to someone invested with authority (whom people are somewhat forced to obey). He suggests that leaders serve three main roles: they are judges, experts and coordinators. While the role of judges and experts are without a doubt important for leadership, I am especially interested in the third role: *leaders as coordinators*. There are some situations in which the behavior of agents following early adopters of a technology or behavior might not be optimal, as in the case of herding [64]. However, very often there is a multiplicity of equilibria (as in the case of different conventions) and the optimal course of action is to coordinate on which equilibrium to choose. Hermalin [65] recognized that even in this case, leaders might have an incentive to select one equilibrium over another, potentially misleading their followers. He analyzed when is it possible to *lead by example*, so the leader is invested in the choice that she is advocating. In the present model there is no ex-ante conflict of interests between agents, as they all agree that more of the public good is better. However, agents can still lead by example, because by contributing they recruit their friends, and it is possible to generate a snowball of social contagion, as we saw in the previous section.

I continue using the concept of Subgame Perfect Equilibrium (SPE). There are two main classes of equilibria which are useful to consider, because they represent the two extremes of the spectrum. At one extreme, **social pressure equilibria** are equilibria where there is a set of agents  $Y$  such that they all fear that others will contribute, and that if they do not contribute they will be punished, and so they end up contributing as well. Therefore, the fear of punishment becomes self-fulfilling, and it is the reason why

<sup>15</sup> Gagnon and Goyal [63] used the concept of the  $q$ -core, defined as the larger subcomponent  $g_q$  of the network such that each individual in  $g_q$  has more than  $q$  friends in  $g_q$ , to characterize the interaction between networks and markets. While at first sight it might seem that the  $q$ -core could characterize contagion in the network, this is not so. For example, in the “chess lattice” of Example 2, the 7-core coincides with the entire network, as every agent has exactly 8 friends in the network. And yet, as we saw, contagion is not guaranteed for  $m^* = 3$ . The intuition behind why concepts such as the  $q$ -core cannot be used to characterize contagion, is because they do not take into account the clustering in the network (or lack thereof).



everybody in  $Y$  contributes. In this case, agents in  $Y$  are not concerned with inducing others to contribute; only with avoiding social punishment. At the other extreme, we have **spearheaded equilibria**, in which there is one agent  $i \in N$ , such that  $i$  contributes even if nobody else would ever contribute. Agent  $i$  does not contribute out of social pressure (after all, if she did not contribute, nobody ever would), but because she wants to generate a contagion in the network, so that many individuals end up contributing, and  $i$  can enjoy the public good generated by such contributions. Because of that,  $i$  is truly a leader, since her reasons to act are to induce others do so.

#### 4.1. Social Pressure Equilibria

I start by analyzing equilibria that demand very little on the part of individuals to contribute, by exploiting the fear of social sanctions. In order to do that, I find a set of agents  $Y$  where they all are willing to contribute, and each agent in  $Y$  contributes because of the fear of social pressure from the rest of agents in  $Y$ . I formalize this intuition in the following definition.

**Definition 4.** A set  $Y$  is  $m^*$ -cohesive if for all  $i \in Y$ ,  $i \in \Pi_{m^*}^\infty(Y \setminus \{i\})$ .

That is,  $Y$  is  $m^*$ -cohesive if all agents who belong to  $Y$  become predisposed when the rest of agents are predisposed (since contagion spreads by having  $m^*$  predisposed friends). Therefore, agents in  $Y$  contribute (even when nobody has done so) because the rest of  $Y$  is also willing to contribute, becoming a self-fulfilling prophecy.

**Definition 5.** A social pressure equilibrium is a Subgame Perfect Equilibrium such that there exists a set  $Y$  in which all agents are predisposed even if nobody else would ever contribute (and therefore agents in  $Y$  contribute whenever they revise their strategy).

**Proposition 3.** If there exists a  $m^*$ -cohesive set  $Y$ , then there is a SPE that is a social pressure equilibrium.

Proposition 3 offers conditions under which a self-fulfilling fear of social pressure leads to individuals in  $Y$  to contribute. This condition is interesting because it expands the contexts in which the public good can be provided: in the case of civil rights movements, the private cost of participating in protests and revolutions can be very high, but even in these cases the conditions in Proposition 3 might still hold, what means that there might be some SPE where contribution happens, even in cases with large private costs. Note that these type of self-fulfilling social pressure equilibria can be inefficient because not only do players not consider the social pressure externalities they impose on others, but the equilibrium is more likely to arise the higher the social pressure parameter  $\psi$  is<sup>16</sup>.

**Example 4** (Cliquish network redux: social pressure equilibrium). Consider again the network from Example 3, that is composed of a collection of cliques, and recall that all agents in a clique are friends with everybody else in that clique. Note that each clique  $C$  with at least  $m^* + 1$  individuals is  $m^*$ -cohesive, as each individual in  $C$  becomes predisposed when everybody else in  $C$  is predisposed, and this means that there is a SPE where agents in clique  $C$  are predisposed even if nobody ever contributes. As we saw in Example 3, this then means that contagion extends to the entire network, as long as there is a path from the initial clique  $C$  to every other clique.

<sup>16</sup> The reasons why contribution happens sometimes and not some other times are difficult to pinpoint (as in every case of multiple equilibria), and can very well be a matter of coordination. In Jimenez-Gomez [66], I analyze the conditions under which players are able to coordinate in revolting against a regime; however it is necessary to assume that each individual has an incentive to participate. Therefore, as mentioned in the Introduction, the present model provides some conditions under which it is incentive compatible to participate in collective action (due to social pressure).

#### 4.2. Spearheaded Equilibria

**Definition 6.** A spearheaded equilibrium is a Subgame Perfect Equilibrium such that there exists an agent  $i$ , for which her best response to every other player never contributing, is to contribute (whenever she can revise her strategy).

I turn now to obtain sufficient conditions for a spearheaded equilibrium to occur. Notice that the key requisite is that there is at least one individual  $i$  who has enough incentives to contribute, even if nobody else would.

**Definition 7.** The *leader contagion threshold* is the minimal  $m^{**}$  such that Equation (3) holds for  $y = 1$  and  $m = m^{**}$ .

In other words, the leader contagion threshold is the minimal number of predisposed friends an agent  $j$  must have, given that she has one friend (the leader) already contributing, so that the condition in Proposition 1 holds, and therefore it is a best response for  $j$  to also contribute (hence becoming predisposed). Given the contagion threshold  $m^*$ , an agent  $i$  needs to calculate how far she can spread contagion, if she wants to become a leader. First, agent  $i$  needs to find which of her friends would become predisposed, if she decided to contribute. Then, she must determine if the contagion generated by herself and the predisposition of her friends, is enough to compensate her private cost. Proposition 4 formalizes this calculus.

**Lemma 1.** Let  $\zeta(x)$  be the expected number of players, out of  $x$  possible agents, that get to play before the game ends; then  $\zeta(x)$  is given by:

$$\zeta(x) = \sum_{m=1}^x \prod_{h=m}^x \frac{h}{h-1 + \frac{1}{Q}}.$$

**Proposition 4.** Suppose that there is an agent  $i$  such that all other agents in the network have a best response of not contributing when nobody else has contributed before, and that

$$\frac{c}{b} < 1 + \zeta(x_i), \quad (4)$$

where  $x_i = |\Pi_{m^*}^\infty(Y_i)|$ , for  $Y_i$  a  $m^{**}$ -cohesive subset of  $N_i$ . Then there is a spearheaded equilibrium in which  $i$  contributes the first time she plays (and this generates contagion to set  $\Pi_{m^*}^\infty(Y_i)$ ).

In other words, agent  $i$  becomes a leader because (1) she predicts that some of her friends will follow suit if given the chance, and (2) the contagion generated by these “early predisposed” agents is enough to make it worthwhile to contribute in the first place for  $i$ , as the expected benefit in the public good by those who contribute will be larger than her private cost<sup>17</sup>. In particular, when agent  $i$  contributes, she lowers the contagion threshold for her friends from  $m^*$  to  $m^{**}$ . If there is a subset of friends of  $i$  that is  $m^{**}$ -cohesive, then  $i$  can count on social pressure to make those friends to contribute, and this sparks the usual contagion in the social network. Social pressure and leadership are therefore completely intertwined in the spearheaded equilibrium, as it combines social pressure (that generates contagion in the  $m^{**}$ -cohesive group, and then further in the entire network), with the cost-benefit analysis that the leader makes, regarding how much contribution to the public good she can induce through her contribution. This also implies that forward looking behavior is fundamental to leadership.

<sup>17</sup> Note that  $\zeta(x_i)$  is between 0 and  $x_i$ : when  $Q \rightarrow 0$ , so that the game ends immediately, nobody besides  $i$  gets to contribute; when  $Q \rightarrow 1$ , so that the game lasts indefinitely, all agents in  $\Pi_{m^*}^\infty(Y_i)$  get to contribute. Therefore, for intermediate values of  $Q$ , the expected number of other individuals who get to contribute is somewhere between 0 and the size of  $\Pi_{m^*}^\infty(Y_i)$ : the more likely the game is to continue, the higher the right hand side of Equation (4), and the more likely a spearheaded equilibrium can exist.

**Example 5** (Hub becomes a leader). Suppose that there is an agent  $L$  that is a hub: the network is composed of a number  $K$  of cliques, and each member of the network (other than  $L$ ) is connected to all members of her clique, and to  $L$ , but to nobody else. When  $L$  chooses to contribute, we can “transform” the scenario into a de facto series of  $K$  isolated cliques, but where each individual has a friend who contributed (agent  $L$ ). When the size of each clique  $C$  is such that  $m^* > |C| > m^{**}$ , there exists no social pressure equilibrium before  $L$  contributes, but after  $L$  contributes there exists a social pressure equilibrium in which every individual ends up contributing. Therefore, if Equation (4) holds for  $x_i = |C| \cdot K$ , then there exists a spearheaded equilibrium in which  $L$  contributes even if nobody would ever do so otherwise.

#### Remarks on Spearheaded Equilibria

While the leader might decide it is worth starting a contagion in the network, can it ever happen that this results in a detrimental outcome for agents in the network? The answer is yes. While the leader finds it in her benefit to contribute and to set contagion in motion (by Equation (4)), she does not internalize the “social pressure externalities” that she imposes on others, that is, the social pressure that will be suffered by those who have friends who contributed, but who did not contribute themselves. Note that the condition in Equation (4) does not directly depend on  $\psi$  (only indirectly through the effect of  $\psi$  on the contagion thresholds  $m^*$  and  $m^{**}$ ); and the higher the social pressure parameter  $\psi$  is, the more likely the equilibrium will be inefficient. However, spearheaded equilibria seem less likely to be inefficient than social pressure equilibria, for the simple reason that the motivation to contribute in a spearheaded equilibrium is to generate sufficient contribution to the public good, what can fully offset the social pressure disutility, whereas in the case of social pressure equilibria they can arise even in cases in which not a substantial contagion in the network will be generated.

Finally, consider contagion from the point of view of a principal who desires to prevent contribution from happening (for example, a dictator who wants to stop a demonstration from happening; or the Montgomery establishment who was against the bus boycott, in the case of Rosa Parks). The principal can impose an extra cost  $\delta > 0$ , only to the first person to contribute. The intuition for this is that, while it is relatively easy for a government or an organized minority to retaliate against a single person, it is very hard to fight against a mass of individuals. For example, Rosa Parks and her husband suffered greatly as a consequence of her becoming a leader of the civil rights movement: they lost their jobs, developed health problems, and received hate call and mail persistently, until they left Montgomery due to this persecution [67]. The key fact to observe is that  $\delta$  affects the cost-to-benefit ratio for the first agent to contribute, which now becomes  $\frac{c+\delta}{b}$ . Note that, for spearheaded equilibria, the left hand side of Equation (4) increases in  $\delta$  and, for  $\delta$  large enough, no individual finds it optimal to become the first to contribute. If the principal has to pay an “intimidation cost”  $\delta$  per player to which it wants to increase the cost, this is an inexpensive way to do so because, in equilibrium, the principal does not even need to pay the cost: the threat of incurring the extra cost  $\delta$  is enough to prevent an agent from becoming a leader. The same holds true for social pressure equilibria. Note that in Equation (3) the left hand side is also increasing in  $\delta$ , and therefore for high enough  $\delta$ , we have that  $m^*$  increases. For  $\delta$  large enough,  $Y$  stops being  $m^*$ -cohesive, for the new value of  $m^*$ . In that case, agents in  $Y$  do not contribute, because the threat of social pressure is not high enough as compared to the private cost  $c + \delta$ . Note that, once again, the principal only needs the threat of the higher cost  $c + \delta$ , in order to stop contagion before it even starts.

#### 4.3. Limited Observability

So far I have implicitly made a somewhat stark assumption: individuals know the network, and moreover they can perfectly observe the actions taken by others, even if they are far away in the network. Instead, we could think that there is **limited observability**: players cannot observe the network, only the actions taken by their friends and when those

friends become predisposed<sup>18</sup>. Hence limited observability implies that the agents do not know what those who are not their friends are choosing, but also know more about their friends (in particular, that they can observe whether they become predisposed).

**Lemma 2.** *The result of Proposition 2 still holds under limited observability, namely given a set  $X$  of predisposed agents, contagion expands to  $\Pi_{m^*}^\infty(X)$ .*

The proof of Lemma 2 is straightforward: because at each step  $k$  of contagion in set  $\Pi_{m^*}^k$ , agents only need to observe their friends in order to become predisposed, then contagion happens identically under limited observability.

**Corollary 1.** *If there exists a  $m^*$ -cohesive set  $Y$ , then there is a SPE where all agents in  $Y$  contribute whenever they play (a social pressure equilibrium), even if there is limited observability.*

Corollary 1 follows from the fact that, for contagion, it is only necessary that agents observe whether their friends become predisposed, not the actions taken by those far away from them in the network. Therefore, through limited observability, contagion spreads because once agents observe enough of their friends predisposed, they become predisposed themselves. On the other hand, for a spearheaded equilibrium under limited observability, we would need one extra condition: that the agent  $i$  who is to become leader actually has perfect observability (i.e., the same level of observability as assumed in previous sections).

**Corollary 2.** *If agent  $i$  has perfect observability, then there exists a spearheaded equilibrium if the conditions of Proposition 4 hold, even if there is limited observability for the rest of the population.*

In summary, the results about leadership are almost intact with limited observability. While the fact that agents can observe when their friends become predisposed is an admittedly non-standard assumption, it is not very far-fetched when thinking about social pressure applications, as agents in social networks are in close contact with their friends in the network and the fact of becoming predisposed could be communicated (this is of course outside of the present model). In summary, the fast contagion in best responses showed to happen in the present model, would survive under a fairly reasonable assumption about limited observability.

## 5. Conclusions

I developed a model of collective action where the agents are forward looking, have concerns for social sanctions, and can only revise their actions stochastically. I analyzed a contagion process through the network, by which agents become predisposed (i.e., are willing to contribute to collective action whenever they can revise their strategy) by best response dynamics. In contrast to earlier models in the literature of social dynamics in which contagion is usually a slow process (that requires several generations of agents best-responding), contagion in the present model happens immediately, due to the forward looking nature of agents. This insight contributes to understanding collective action and public good provision (and more generally the evolution of cooperation in humans), by showing that forward looking agents are even more likely to cooperate. It also provides microfoundations to a long-standing literature that assumes that the decision to participate in collective action and revolutions can be modeled as a coordination game [17–21].

I also analyzed leadership: when an agent chooses to contribute even when nobody has contributed so far. One condition that suffices for contribution happening, is that the

<sup>18</sup> Formally, we can assume that after each period, all agents observe the actions of their friends, and they become predisposed if they will contribute whenever they can revise their strategy. If any agent that was not predisposed has become predisposed, then again some agents might become predisposed, and so forth. This process continues until no agent becomes predisposed, and at that moment the next period begins. Note that I am assuming that agents still have knowledge of the network, even if they cannot observe the actions (or predisposition state) of those who are not their friends.

expected gain from contributions by those affected by the contagion is larger than the private cost of contributing. This is the case when a single agent can use her influence on her friends, who in turn use their influence on theirs, to generate a wave of contributions in the network, that eventually compensates for the private cost of contributing in the first place. Forward looking behavior is critical: it is because agents are able to foresee the contagion that they are capable of generating, that they decide to lead by contributing. Under less demanding conditions there can also be contribution: in these cases, agents start contributing not because of desire of generating contagion, but out of a self-fulfilling fear of being socially sanctioned.

This paper contributes to the literature in social dynamics by considering a population fully composed of forward looking rational agents, through a simple model that can be easily applied to many other contexts. Indeed, the strength of the model lies in its simplicity: the results can be obtained without resorting to convoluted proofs or arguments. However, to achieve such simple setup, a number of assumptions had to be made, including the exogeneity of the network and of social pressure. Those assumptions were made to show the existence and properties of fast contagion due to best-responses for a relevant class of games. However, a promising avenue of future research consists on improving these assumptions and finding weaker conditions under which meaningful results could be derived. Several extensions could be developed for the baseline model. I have assumed homogeneity in the benefits, costs, and punishments. Introducing heterogeneity would generate new interesting predictions; in particular, analyzing how heterogeneity in the parameters interacts with homophily (the tendency of people to have friends like themselves), when both heterogeneity and homophily take place along the same dimension (i.e., agents with lower cost of contribution tend to be friends with each other). This could be modeled using scale-free networks (in which the degree distribution has fat tails, and that better capture real-world networks such as the Internet or Twitter, [68]) and that have already been used to analyze the Prisoner's Dilemma with heterogeneous investments [69,70]. This would open up new questions, such as whether the network would be robust to a government trying to stifle potential leaders, for example by removing hubs from the network [71]. A related question is how dispersion in those variables, as measured by Second Order Stochastic Dominance, affects contagion and leadership. For example, it is not clear ex-ante that more dispersion in cost would lead unambiguously to more contagion, or more leadership.

In conclusion, this paper offers a simple explanation for why individuals participate in collective action in the presence of social pressure. The conclusions on this paper can be incorporated to the literature on political revolutions as a justification of why (and how) it is incentive compatible, under certain circumstances, for individuals to participate in collective action, giving a more solid microfoundation to reduced-form models of revolutions. More generally, the paper offers a simple and tractable model that can be applied in any area of economics, to the empirically ubiquitous finding that social pressure generates public good in social networks.

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## Appendix A

**Proof of Proposition 1.** We are interested in sufficient conditions, and therefore I will consider a lower bound for the payoff of contributing, and an upper bound on the payoff for not contributing. The lower bound on the payoff for an agent for contributing, is given by  $b - c$ . The upper bound on expected payoff for not contributing can be calculated from the different possibilities of punishment, namely the different events that a number of the agent's predisposed friends get to contribute before the game ends *and* before the agent can revise her strategy. In particular, let  $P(x|m)$  be the probability that, out of  $m$  predisposed friends of agent  $i$ , exactly  $x \leq m$  revise their strategy (hence contributing), before the game ends (but agent  $i$  does not revise her strategy). Note that for  $x > 1$ :

$$P(x|m) = q \left[ \frac{m}{n} P(x-1|m-1) + \frac{n-m-1}{n} P(x|m) \right],$$

and therefore

$$P(x|m) = \frac{m}{n(q-1) + m + 1} P(x-1|m-1).$$

For  $x = 0$ , we have

$$P(0|m) = 1 - q + q \left[ \frac{n-m-1}{n} P(0|m) \right] \implies P(0|m) = \frac{1-q}{1 - q \frac{n-m-1}{n}}.$$

Equipped with these probabilities, we can now compute an upper bound on the expected payoff from not contributing, which is given by:

$$- \psi b \sum_{x=0}^m P(x|m)(y+x). \quad (\text{A1})$$

Therefore, a sufficient condition for the to find optimal to contribute (and hence becoming predisposed) is that  $b - c$  is larger than the expression in Equation (A1), that is, when

$$\frac{c}{b} < 1 + \psi \sum_{x=0}^m P(x|m)(y+x).$$

□

**Proof of Proposition 2.** The proof is by induction. By definition,  $X$  is a set of predisposed agents, what proves the case  $k = 0$ . Now, suppose the statement is true for  $k$ , what means that all individuals in  $\Pi_{m^*}^k(X)$  are predisposed. Then, by the definition of  $m^*$ , all agents with at least  $m^*$  friends in the set  $\Pi_{m^*}^k(X)$  will also become predisposed, what implies that individuals in  $\Pi_{m^*}^{k+1}(X) = \Pi_{m^*}^k(X) \cup \{i \in N : |N_i \cap \Pi_{m^*}^k(X)| \geq m^*\}$  become predisposed. □

**Proof of Proposition 3.** Let  $\sigma$  be the strategy profile where agents in  $Y$  always contribute (and all  $j \notin Y$  best respond to  $N \setminus Y$ ). Consider an agent  $i \in Y$ . Because agents in  $Y \setminus \{i\}$  are predisposed, by Proposition 2, all agents in  $\Pi_{m^*}^\infty(Y \setminus \{i\})$ , become predisposed. But since  $Y$  is  $m^*$ -cohesive, that means that  $i \in \Pi_{m^*}^\infty(Y \setminus \{i\})$ , and therefore  $i$  becomes predisposed, so it is a best response for  $i$  to contribute. □

**Proof of Lemma 1.** Note that  $\zeta(x)$  is given recursively by:

$$\zeta(x) = q(n) \left[ \frac{x}{n} (\zeta(x-1) + 1) + \frac{n-x}{n} \zeta(x) \right],$$

and  $\zeta(1) = Q$ , by the definition of  $Q$ . Solving for  $\zeta(x)$ , and using the formula for  $q(n)$  as a function of  $Q$ , we have that for  $x > 1$ ,  $\zeta(x) = \frac{x[1+\zeta(x-1)]}{x-1+\frac{1}{Q}}$ . I will show now that  $\zeta$  is given by:

$$\zeta(x) = \sum_{m=1}^x \prod_{l=m}^x \frac{l}{l-1 + \frac{1}{Q}} \quad (\text{A2})$$

The proof is by induction. For  $x = 1$ ,  $\zeta(1) = Q$ , and from Equation (A2), we find the same value. Now, for the induction step, suppose that Equation (A2) holds up to  $x$ , and I will prove it for  $x + 1$ :

$$\begin{aligned} \zeta(x+1) &= \frac{(x+1)[1 + \zeta(x)]}{x + \frac{1}{Q}} = \frac{x+1}{x + \frac{1}{Q}} \left[ 1 + \sum_{m=1}^x \prod_{l=m}^x \frac{l}{l-1 + \frac{1}{Q}} \right] = \\ &= \frac{x+1}{x + \frac{1}{Q}} + \sum_{m=1}^x \frac{x+1}{x + \frac{1}{Q}} \prod_{l=m}^x \frac{l}{l-1 + \frac{1}{Q}} = \frac{x+1}{x + \frac{1}{Q}} + \sum_{m=1}^x \prod_{l=m}^{x+1} \frac{l}{l-1 + \frac{1}{Q}} = \sum_{m=1}^{x+1} \prod_{l=m}^{x+1} \frac{l}{l-1 + \frac{1}{Q}}. \end{aligned}$$

And this concludes the proof.  $\square$

**Proof of Proposition 4.** First, conditional on agent  $i$  contributing, note that an argument identical to that in Proposition 3 implies that all agents in  $Y_i$  become predisposed, for  $Y_i$  a  $m^{**}$ -cohesive subset of  $N_i$  (the argument is identical, but  $m^*$  is replaced by  $m^{**}$ , since all friends of  $i$  would have at least one friend who contributed). Then, Proposition 3 implies that all agents in set  $\Pi_{m^*}^\infty(Y_i)$  also become predisposed. Hence, agent  $i$  knows that if she contributes, the expected contribution in the network is at least  $b(1 + \zeta(x_i))$  for  $x_i = |\Pi_{m^*}^\infty(Y_i)|$ , and if she does not contribute her expected payoff is zero (because all other agents would never contribute). Therefore, she finds it optimal to contribute whenever Equation (4) holds, and hence contributing is a best response to nobody ever contributing.  $\square$

**Proof of Corollary 1.** The proof follows from Proposition 3 and Lemma 2: given  $i \in Y$  and an initial set  $Y \setminus \{i\}$  of predisposed agents, each step of contagion in  $\Pi_{m^*}^k(Y \setminus \{i\})$  happens under limited observability of actions, and therefore  $i \in \Pi_{m^*}^\infty(Y \setminus \{i\})$ , so  $i$  is predisposed, and this holds for all  $i \in Y$ .  $\square$

**Proof of Corollary 2.** The proof follows from Proposition 4 and Lemma 2. Note that from Lemma 2, we have that once agent  $i$  contributes, contagion first extends to set  $Y_i$  (defined as a  $m^{**}$ -cohesive set of friends of  $i$ ), and then to set  $\Pi_{m^*}^\infty(Y_i)$ , even under limited observability. Therefore, the only condition that is necessary is that agent  $i$  contributes, and if she has perfect observability, then she can perform the cost-benefit analysis in Equation (4), and it is a best response for her to contribute whenever Equation (4) holds.  $\square$

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