Sustainability **2014**, *6*, 4339-4354; doi:10.3390/su6074339

OPEN ACCESS

sustainability

ISSN 2071-1050

www.mdpi.com/journal/sustainability

Article

The Equilibrium Decisions in a Two-Echelon Supply Chain under Price and Service Competition

Xiaonan Han [1], Xiaochen Sun [1,*] and Yancong Zhou [2]

[1] Department of Mathematics,Tianjin University, Tianjin 300072, China;
E-Mail:hanxiaonan1620@126.com

[2] School of Information Engineering, Tianjin University of Commerce, Tianjin 300134, China;
E-Mail: zycong78@126.com

* Author to whom correspondence should be addressed; E-Mail: xcsun@tju.edu.cn;
Tel.:+86-13920432116.

Received: 24 March 2014; in revised form: 1 July 2014 / Accepted: 8 July 2014 /
Published: 14 July 2014

Abstract: This article studies a supply chain composed of a manufacturer and two competing retailers. The manufacturer produces two substitutable products and offers respective service levels to customers who buy one of the two products. Each retailer can only order one kind of product from the manufacturer, and then sell them to the market at a certain sale price. The demands for two products are influenced not only by the service levels the manufacturer provides, but also the sales prices of the two products. Furthermore, we investigate the equilibrium behavior of members in the supply chain with the aid of the Stackelberg game, and discover a number of insights concerning some important parameters. Finally, Numerical analysis is presented to validate our theoretical results and compare channel performances.

Keywords: supply chain; service competition; substitutable products; Stackelberg game

---

1. Introduction

Recently, how companies compete with their rivals and what factors affect the companies' behavior in competition have become a hot topic focused on by managers. Pricing is a significant business strategy and competing firms often engage in a price war to attract customers [1]. However, as the living standard of people is improving with the development of society, people become more and more sensitive to non-price factors they could enjoy rather than a single price attribute. Consequently, the competition of firms becomes increasingly keen and the form of competition is evolving from the single price competition to other important factors, such as service accompanying the products. For example, SAMSUNG Corporation and its competitor Apple Inc. both offer services for phones to customers, in order to motivate customers' willingness to buy. It is easy to understand one certain kind of products' market demand is positively influenced by the service the company offers however negatively blocked by its rival's service. In addition, largely due to the rapid development of technology, the life cycle of a product is shorter and shorter, making the co-existence of products of two different generations prevalent, especially in the electrical industry. The manufacturer usually provides different service offerings for the new and old generation products. Evidently, the products of two different generations can impact upon each other in service level. Besides, in auto industry, financial services such as auto loan, insurance, and maintenance service play an important role in selecting a brand for customers [1]. Therefore, only by putting effort into strengthening products' service level, can a company become more attractive to customers. Such services often include information about how to install or use the product, maintenance service, or warranty offerings. Each of these can improve the consumers' perceived value of the product. However, unlike other dimensions of product quality, the services that are included in a product bundle can often be provided by either a manufacturer or by a dealer. To the best of our knowledge, most of previous literature examining service competition premise the services are provided by the retailer. Here, we assume the services are performed by the manufacturer, which is common in the electronics industry. A steep service level puts a service cost burden on the manufacturer, while a too low service level can divert more customers to other products. Hence, what a manufacturer should do is to choose the optimal service level for his products.

In this study, we incorporate both price competition and service competition into our supply chain model of a manufacturer and two competing retailers. The manufacturer produces two substitutable products, which influence on each other interactively, and provide service offerings for the two products. Each retailer can only buy one kind of product from the manufacturer. A point to note is that the products wholesaled to the first retailer are different from those wholesaled to another retailer. While each retailer determines its own selling price, the manufacturer controls the two products' service levels. The purpose of this research is to provide insights about the equilibrium behavior of the supply chain's members under the framework of price and service competition. There exists a Stackelberg game between the manufacturer and two retailers, and we examine the drivers of each retailer's price strategy and the manufacturer's service strategy. It is proved that the intensity of service competition plays a key role in the members' equilibrium decisions, if some specific limitations are imposed. It is also understood that if the manufacture benefits more from one kind of products than does from another kind of products, the manufacturer will be likely to offer a higher service level for the first kind of product. These results are well in line with our intuition.

We first discuss the issue of Stackelberg game in supply chain in two or multiple echelon settings. We will then focus on efforts to introduce some literature aiming at price and service competition. There is a growing amount of research in relation to Stackelberg game problems in supply chain. The goals of every chain members are to maximize their own profit, and all of them could make their decisions independently. So there exists game playing between chain members. What is most commonly utilized is the Stackelberg game. Choi [2] studies a Stackelberg game model with two competing manufacturers and a common retailer that sells both manufacturers' products. The single retailer acts as the Stackelberg-leader. This paper also explores three noncooperative games of different power structures between the two manufacturers and the retailer. Yan *et al.* [3] focus on cooperative advertising and the effect it exerts on the dual channel supply chain. They also obtain equilibrium pricing policies under two different competitive scenarios: Bertrand and Stackelberg equilibrium. Furthermore, they also compare the profit gains under these two marketing games. Liu and Xu [4] explore the pricing problem in a fuzzy supply chain with a manufacturer and two competitive retailers. The manufacturer playing as a leader determines the wholesale price, and the retailers acting as the followers choose their sale prices independently. Many other papers have also addressed the Stackelberg game problem in the supply chain, such as [5,6]. However, all of them pay little attention to depicting price and service competition, and they seldom tell us how the price and service affects market demand.

More recently, studies have begun to take price and service competition into consideration, largely because selling price and service level of products are exerting a significant influence on customers' purchasing behavior. Hsieh *et al.* [7] study coordination mechanisms in a supply chain comprised of two suppliers with capacity uncertainties selling differential yet substitutable products through a common retailer who faces random demand of these two products which incorporate price competition. The analytical and empirical results in [8] offer guidelines to e-tailers on how to price their products and decide their service offerings considering price competition. Both of the above do not take service as a influencing factor into the model. Lyer [9] analyzes how manufacturers should coordinate distribution channels when retailers compete in price as well as important nonprice factors such as the provision of product information, free repair, faster check-out, or after-sales service. This paper also shows that for relatively high-ticket items retailers tend to be excessively biased towards competing in the provision of retail services. Besides, the basic model is also extended to consider the effect of upstream competition between manufacturers. Tsay and Agrawal [10] explore a supply chain with two competitive retailers with a common manufacturer in which both the retailers provide products as well as service to customers. However, in our paper the services for products are provided by the manufacturer. Xia and Gilbert [11] concentrate on a supply chain model with two substitutable products in which the product service levels are offered by the manufacturer and consider a game that is played between the manufacturer and the dealer. Xiao and Yang [1] develop a price-service competition model of two supply chains to investigate the optimal decisions of players under demand uncertainty. Wu [12] considers both price and service competition between new and remanufactured products in a two-echelon supply chain. The supply chain consists of two manufacturers, one produces the new products and another produces remanufactured products, and a common retailer. And the services accompanying the products are rendered by the two manufacturers, respectively. Zhao *et al.* [13] explore a distribution system in which two competitive manufacturers supply two substitutable products to one common retailer, who in turn sells them to the end consumers. The effects of price and service level on product demand are also considered in their model.

Our paper differs mainly from the above articles in the following aspects. Firstly, unlike those models assuming that two competitive retailers sell the same products, it is supposed that the two retailers sell substitutable products in our paper. This assumption reasonably explains why the manufacturer render distinctive service levels for the two products. Secondly, we incorporate both price and service competition into our model, which is applicable in the scenario we consider. Last but not the least, what is different from other literature is that in our study the services are provided by the manufacturer rather than the retailers.

The rest of the paper is organized as follows. A general model is formulated in Section 2. In Section 3 the existence and uniqueness of the equilibrium decisions are demonstrated and we obtain the explicit expressions of the equilibrium strategies, and we further perform analysis on the property of the equilibrium decisions. Numerical analysis is presented in Section 4. Finally, conclusions and further directions are addressed in Section 5.

2. Model Formulation

In this paper, we discuss a two echelon supply chain consisting of a single manufacturer, which is denoted by M, and two competitive retailers denoted by $R1$ and $R2$. Now, in a customer-oriented retail market, the manufacturer constantly develops various products in order to satisfy a variety of customer needs. For the sake of simplicity, in this paper, we assume that the manufacturer produces two substitutable products and every retailer can choose only one kind of product from the two products available. The two substitutable products may differ in some dimensions such as color or style. They have distinctive production cost and wholesale price, however, they are functionally the same. Customers hold different perceptions about the two products, and make buying choices between them based on personal preference. Actually, the end consumers' perception of value and their purchase decisions are not influenced exclusively by the item's selling price, but also the service level that accompanies it. Here, service is taken to broadly represent all forms of demand-enhancing effort, including customer service before and after the sale, in-store promotions, advertising and warranty offerings [10]. Many such services can be provided either by the manufacturer or they can be delegated to the dealer. A scenario where the manufacturer engages in offering service levels is considered here. In this model, the manufacturer is the Stackelberg leader and the two retailers are the followers. The manufacturer who could observe the retailers' decisions possesses an advantageous position, but he can not control what the retailers choose to do. The two retailers make their own decisions independently. The effects of variation of time on chain members' decisions are not taken into consideration. The sequence of events is as follows. Firstly, the manufacturer chooses best service levels to maximize his profit before the selling season; then the two retailers achieve the Nash equilibrium in selling prices of two products to maximize their own profit based on the service levels chosen by the manufacturer. We assume all supply chain members have access to the same and complete information when optimizing their objective functions.

2.1. The General Optimal Model

Let d_1 and d_2 respectively denote demand for products 1 and 2. As popularly used in other literature such as [10,12] on price and service competition, here d_1 and d_2 are thought of as linear functions about prices and service levels. Thus, the demand functions could be described by incorporating price and service competition as follows.

The demand function of product 1:

$$d_1(p_1, p_2, s_1, s_2) = \alpha_1 - \beta_p p_1 + \gamma_p(p_2 - p_1) + \beta_s s_1 - \gamma_s(s_2 - s_1) \quad (1)$$

The demand function of product 2:

$$d_2(p_1, p_2, s_1, s_2) = \alpha_2 - \beta_p p_2 + \gamma_p(p_1 - p_2) + \beta_s s_2 - \gamma_s(s_1 - s_2) \quad (2)$$

where α_1, α_2, β_p, β_s, γ_p and $\gamma_s \geq 0$. α_i describes the base-case potential market size for each product. Specifically, it describes the demand for product i when both products' prices are 0 and there is no investment in service levels. β_p and β_s measure the responsiveness of market demand to its own price and service level, respectively. γ_p and γ_s respectively depict the intensity of competition between the two products with regards to pricing and service levels. Today customers are provided more chance to choose freely, so more and more customers are likely to make a comparison between similar products before they make a decision to buy. So the intensity of service competition exerts a more important influence on customers' willingness-to-buy than itself service level does. Therefore, $\gamma_s > \beta_s$ is assumed in this article. Apparently, the market demand of each product is an increasing function of his rival's retail price and his own service level, but a decreasing function of his own retail price and his rival's service level. As expressed in [10,12], if all else parameters are held equal, cutting p_i by one unit will appeal to $\beta_p + \gamma_p$ more customers: β_p is the additional customers induced from a decrease in retail price of product i, and the remaining γ_p customers are diverted from the rival product. It is obviously expounded by $\frac{dd_i}{dp_i} = -(\beta_p + \gamma_p)$ and $\frac{dd_j}{dp_i} = \gamma_p$ $(i \neq j)$. Hence, a higher value of γ_p leads to fierce price competition between two competitive products, and γ_s has a similar connotation for the service competition. More details about the parameters mentioned above are described in [10,12].

We premise that the manufacturer is fully aware of the market demand for products 1 and 2, that is to say, the manufacturer produces the very amount of products 1 and 2. Therefore, given the above demand functions d_1 and d_2, the manufacturer's profit function could be achieved. Obviously, in our model the manufacturer bears the production cost and service cost. We denote the service cost by $\frac{ms_i^2}{2}(i = 1 \; or \; 2)$, where the quadratic form suggests diminishing returns on such expenditures. And m is the ultimate cost of service as in [10]. Similar approaches to modeling service effort have been used in a large amount of other literature. Hence, the manufacturer's profit function is

$$\Pi_M(s_1, s_2) = (w_1 - c_1)d_1(p_1, p_2, s_1, s_2) + (w_2 - c_2)d_2(p_1, p_2, s_1, s_2) - \frac{ms_1^2}{2} - \frac{ms_2^2}{2} \quad (3)$$

w_1 and w_2 respectively denote the wholesale prices and c_1 and c_2 respectively the production costs of products 1 and 2.

We can evidently observe that the wholesale prices have influence on the manufacturer's profit and, in turn, the two retailers' expected profits. Then the manufacturer would engage in a game for determining the wholesale prices of the two competitive products. However, such equilibrium values of the wholesale prices are affected by many unquantifiable factors, such as past relationship with the retailer, or the eagerness of the retailers in getting products. Consequently, the wholesale prices of products are unattainable numerically and the manufacturer has to roughly estimate his wholesale price. Therefore, it is assumed here that the wholesale prices of two products are both predetermined, which is very similar to [7].

Then, the profit functions of retailer $R1$ and $R2$ can be formulated by the following:

$$\Pi_{R1}(p_1) = (p_1 - w_1)d_1(p_1, p_2, s_1, s_2) \tag{4}$$

$$\Pi_{R2}(p_2) = (p_2 - w_2)d_2(p_1, p_2, s_1, s_2) \tag{5}$$

Based on the above Functions (3)–(5), the general optimization model can be described as follows:

$$Max\Pi_M(s_1, s_2) = (w_1 - c_1)d_1(p_1, p_2, s_1, s_2) + (w_2 - c_2)d_2(p_1, p_2, s_1, s_2) - \frac{ms_1^2}{2} - \frac{ms_2^2}{2}$$

subject to:

$$\begin{cases} Max\Pi_{R1}(p_1) = (p_1 - w_1)d_1(p_1, p_2, s_1, s_2) \\ Max\Pi_{R2}(p_2) = (p_2 - w_2)d_2(p_1, p_2, s_1, s_2) \end{cases} \tag{6}$$

We can see that, based on the analysis stated above, in this model the manufacturer determines the service levels s_1 and s_2, and the two retailers set the retail prices p_1 and p_2, respectively.

3. Results and Discussion

We now derive chain members' optimal strategies by maximizing their profits in the above general optimization model. The following theorems characterize chain members' profit functions with respect to their decisions.

Theorem 1. *For any given service levels s_1 and s_2: (i) the profit function $\Pi_{R1}(p_1)$ is concave with respect to sales price p_1 of product 1; (ii) the profit function $\Pi_{R2}(p_2)$ is concave with respect to sales price p_2 of product 2.*

Proof. Consider the following derivatives:

$$\begin{aligned} \frac{\partial \Pi_{R1}(p_1)}{\partial p_1} &= d_1 - (p_1 - w_1)(\beta_p + \gamma_p) \\ \frac{\partial^2 \Pi_{R1}(p_1)}{\partial p_1^2} &= -2(\beta_p + \gamma_p) \\ \frac{\partial \Pi_{R2}(p_2)}{\partial p_2} &= d_2 - (p_2 - w_2)(\beta_p + \gamma_p) \\ \frac{\partial^2 \Pi_{R2}(p_2)}{\partial p_2^2} &= -2(\beta_p + \gamma_p) \end{aligned}$$

Because $\frac{\partial^2 \Pi_{R1}(p_1)}{\partial p_1^2} = \frac{\partial^2 \Pi_{R2}(p_2)}{\partial p_2^2} = -2(\beta_p + \gamma_p) < 0$, $\Pi_{R1}(p_1)$ is concave with respect to p_1 and $\Pi_{R2}(p_2)$ is concave with respect to p_2, that is to say, there do exist sales prices p_1' and p_2' respectively maximizing $\Pi_{R1}(p_1)$ and $\Pi_{R2}(p_2)$. □

In order to obtain equilibrium decisions, the backward method is usually employed to solve the Stackelberg game problem. First, we solve the two retailers' optimal response functions under the condition that the manufacturer's decisions are given. Then we solve the equilibrium decisions of the manufacturer after the two retailers' optimal response functions are obtained. All optimal response functions are denoted by the superscript "′" and let the superscript "$*$" denote the equilibrium decisions.

Proposition 1. *For any given s_1 and s_2, the two retailers' best response functions satisfy equations*

$$p_1^{'} = \frac{2\Lambda_p(\Lambda_p w_1 + \alpha_1 + \Lambda_s s_1 - \gamma_s s_2) + \gamma_p(\Lambda_p w_2 + \alpha_2 + \Lambda_s s_2 - \gamma_s s_1)}{4\Lambda_p^2 - \gamma_p^2} \tag{7}$$

$$p_2^{'} = \frac{2\Lambda_p(\Lambda_p w_2 + \alpha_2 + \Lambda_s s_2 - \gamma_s s_1) + \gamma_p(\Lambda_p w_1 + \alpha_1 + \Lambda_s s_1 - \gamma_s s_2)}{4\Lambda_p^2 - \gamma_p^2} \tag{8}$$

It can be easily proved by solving the first conditions $\frac{\partial \Pi_{R1}(p_1)}{\partial p_1} = 0$ and $\frac{\partial \Pi_{R2}(p_2)}{\partial p_2} = 0$. Note that $\Lambda_p \equiv \beta_p + \gamma_p$ is the overall effect of the product's price on its demand, and $\Lambda_s \equiv \beta_s + \gamma_s$ represents the overall effect of service on demand. We also could refer to [9] for further information. Paying attention to the impact of service levels on the two retailers' best response functions, we derive the first derivatives of $p_1^{'}$ and $p_2^{'}$ with respect to the service levels, *i.e.*, $\frac{\partial p_1'}{\partial s_1} = \frac{\partial p_2'}{\partial s_2} = \frac{2\Lambda_p\Lambda_s - \gamma_s\gamma_p}{4\Lambda_p^2 - \gamma_p^2} > 0$ and $\frac{\partial p_1'}{\partial s_2} = \frac{\partial p_2'}{\partial s_1} = \frac{\gamma_p\Lambda_s - 2\Lambda_p\gamma_s}{4\Lambda_p^2 - \gamma_p^2}$. We can easily see that, the better the service of product i is, the higher is the sales price of product i charged by the retailer Ri ($i = 1, 2$). However, the effect of service level of product i on the sales price of product k ($k \neq i$) is not very explicit.

Theorem 2. *The manufacturer's profit function $\Pi_M(s_1, s_2|p_1', p_2')$ is concave with respect to the service levels s_1 and s_2.*

Proof. It is known

$$\Pi_M(s_1, s_2|p_1', p_2') = (w_1 - c_1)d_1(p_1', p_2', s_1, s_2) + (w_2 - c_2)d_2(p_1', p_2', s_1, s_2) - \frac{ms_1^2}{2} - \frac{ms_2^2}{2}$$

From Equations (7) and (8), we get

$$\begin{aligned} \frac{\partial p_1'}{\partial s_1} = \frac{\partial p_2'}{\partial s_2} &= \frac{2\Lambda_p\Lambda_s - \gamma_s\gamma_p}{4\Lambda_p^2 - \gamma_p^2} \\ \frac{\partial p_1'}{\partial s_2} = \frac{\partial p_2'}{\partial s_1} &= \frac{\gamma_p\Lambda_s - 2\Lambda_p\gamma_s}{4\Lambda_p^2 - \gamma_p^2} \end{aligned}$$

Furthermore, the first condition of $\Pi_M(s_1, s_2|p_1', p_2')$ is obtained

$$\begin{aligned} \frac{\partial \Pi_M(s_1, s_2|p_1', p_2')}{\partial s_1} &= (w_1 - c_1)(\Lambda_s - \Lambda_p\frac{\partial p_1'}{\partial s_1} + \gamma_p\frac{\partial p_2'}{\partial s_1}) \\ &+ (w_2 - c_2)(-\gamma_s - \Lambda_p\frac{\partial p_2'}{\partial s_1} + \gamma_p\frac{\partial p_1'}{\partial s_1}) - ms_1 \\ \frac{\partial \Pi_M(s_1, s_2|p_1', p_2')}{\partial s_2} &= (w_1 - c_1)(-\gamma_s - \Lambda_p\frac{\partial p_1'}{\partial s_2} + \gamma_p\frac{\partial p_2'}{\partial s_2}) \\ &+ (w_2 - c_2)(\Lambda_s - \Lambda_p\frac{\partial p_2'}{\partial s_2} + \gamma_p\frac{\partial p_1'}{\partial s_2}) - ms_2 \end{aligned}$$

Then we have

$$\begin{aligned} \frac{\partial^2 \Pi_M(s_1, s_2|p_1', p_2')}{\partial s_1^2} &= \frac{\partial^2 \Pi_M(s_1, s_2|p_1', p_2')}{\partial s_2^2} = -m \\ \frac{\partial^2 \Pi_M(s_1, s_2|p_1', p_2')}{\partial s_1 \partial s_2} &= 0 \end{aligned}$$

Consequently, $\Pi_M(s_1, s_2|p_1', p_2')$ is concave with respect to the service levels s_1 and s_2. There does exist s_1' and s_2' maximizing $\Pi_n^M(w_n|p_o', p_n')$. $\square$

With the best response functions for selling prices in Equations (7) and (8), the manufacturer's best response in terms of service levels of products 1 and 2 can be achieved by solving $s_1^*, s_2^* \epsilon$ arg $max_{s_1,s_2} \Pi_M(s_1, s_2|p_1', p_2')$. Let $\partial\Pi_M(s_1, s_2|p_1', p_2')/\partial s_1 = 0$ and $\partial\Pi_M(s_1, s_2|p_1', p_2')/\partial s_2 = 0$, we get the equilibrium service levels s_1^* and s_2^*:

$$s_1^* = \frac{\Lambda_p[(w_1 - c_1)(\gamma_p\gamma_s - 2\Lambda_p\Lambda_s) + (w_2 - c_2)(2\Lambda_p\gamma_s - \gamma_p\Lambda_s)]}{m(\gamma_p^2 - 4\Lambda_p^2)} \quad (9)$$

$$s_2^* = \frac{\Lambda_p[(w_1 - c_1)(2\Lambda_p\gamma_s - \gamma_p\Lambda_s) + (w_2 - c_2)(\gamma_p\gamma_s - 2\Lambda_p\Lambda_s)]}{m(\gamma_p^2 - 4\Lambda_p^2)} \quad (10)$$

As seen from the above equations, the equilibrium service levels s_1^* and s_1^* are both decreasing in the ultimate cost of service m. As the ultimate cost of service m increases, the cost caused by offering service to customers increases too. So the manufacturer will be reluctant to maintain higher service levels. We can derive the equilibrium retail prices p_1^* and p_2^* by taking the expressions of s_1^* and s_1^* into Equations (7) and (8).

3.1. Comparisons between the Equilibrium Decisions

Proposition 2. *If $w_2 - c_2 > w_1 - c_1$, $s_1^* < s_2^*$; otherwise, $s_1^* \geq s_2^*$.*

Proof. It is easy to know $2\Lambda_p\Lambda_s + 2\Lambda_p\gamma_s - \gamma_p\gamma_s - \gamma_p\Lambda_s > 0$. Based on the Equations (9) and (10), if $w_2 - c_2 > w_1 - c_1$, we obtain

$$\begin{aligned} s_1^* - s_2^* &= \frac{\Lambda_p}{m(\gamma_p^2 - 4\Lambda_p^2)}[(w_2 - c_2) - (w_1 - c_1)](2\Lambda_p\Lambda_s + 2\Lambda_p\gamma_s - \gamma_p\gamma_s - \gamma_p\Lambda_s) \\ &< 0 \end{aligned}$$

Namely, $s_1^* < s_2^*$. And we could also lightly get $s_1^* \geq s_2^*$, if $w_2 - c_2 \leq w_1 - c_1$. □

$w_2 - c_2 > w_1 - c_1$ is a sufficient and necessary condition to guarantee $s_1^* < s_2^*$. That is to say, if the manufacture benefits more from unit product 2 than does from unit product 1, it is more desirable for him to offer a higher service level for product 2. Conversely, in the case that wholesaling one product 2 to retailer $R2$ bring less benefit to the manufacturer than wholesaling one product 1 to retailer $R1$ does, the manufacturer prefers to render a higher service level for product 1.

Proposition 3. *(i) If $w_2 - c_2 > w_1 - c_1$ and $\Lambda_p(c_1 - c_2) < \alpha_2 - \alpha_1$, $p_1^* < p_2^*$. (ii) If $w_2 - c_2 < w_1 - c_1$ and $\Lambda_p(c_1 - c_2) > \alpha_2 - \alpha_1$, $p_1^* > p_2^*$.*

Proof. Based on Equations (7) and (8), we have

$$p_1^* - p_2^* = \frac{\Lambda_p(w_1 - w_2) + (\alpha_1 - \alpha_2) + (\Lambda_s + \gamma_s)(s_1^* - s_2^*)}{2\Lambda_p + \gamma_p}$$

If $w_2 - c_2 > w_1 - c_1$ and $\Lambda_p(c_1 - c_2) < \alpha_2 - \alpha_1$, $\Lambda_p(w_1 - w_2) + (\alpha_1 - \alpha_2) < \Lambda_p(c_1 - c_2) + (\alpha_1 - \alpha_2) < 0$ could be proved. And with the aid of Proposition 2, here we know $s_1^* < s_2^*$. Hence, it is obtained $\frac{\Lambda_p(w_1 - w_2) + (\alpha_1 - \alpha_2) + (\Lambda_s + \gamma_s)(s_1^* - s_2^*)}{2\Lambda_p + \gamma_p} < 0$, namely, $p_1^* < p_2^*$. (ii) could be proved in the same way. □

As shown in Proposition 2, if $w_2 - c_2 > w_1 - c_1$ is set, the manufacturer prefers to offer a higher service level of product 2, which is instrumental to enhancing demand for product 2. So the retailer $R2$ are endowed with more room to raise the product 2's selling price. (ii) can also be expounded in the same way. However, which of the equilibrium prices of product 1 and 2 is larger on the condition $w_2 - c_2 > w_1 - c_1$ and $\Lambda_p(c_1 - c_2) > \alpha_2 - \alpha_1$, or $w_2 - c_2 < w_1 - c_1$ and $\Lambda_p(c_1 - c_2) < \alpha_2 - \alpha_1$, is difficult to describe exactly. We now set $w_1 = 15$, $c_1 = 9$, $w_2 = 10$, $c_2 = 2$, $\alpha_1 = 50$, $\alpha_2 = 30$, $r_p = 1$, $\beta_p = 1/2$, $\gamma_s = 3$, $\beta_s = 2$, $m = 2$. In this case $w_2 - c_2 > w_1 - c_1$ and $\Lambda_p(c_1 - c_2) > \alpha_2 - \alpha_1$ are both satisfied, and we get $p_1^* > p_2^*$. Then we reset $\alpha_1 = 40$, $\alpha_2 = 30$ and other parameters are held equal, we obtain $p_1^* < p_2^*$ though the requirements $w_2 - c_2 > w_1 - c_1$ and $\Lambda_p(c_1 - c_2) > \alpha_2 - \alpha_1$ are met too.

3.2. The Effects of Some Parameters on the Equilibrium Decisions

Proposition 4. *(i) The equilibrium service level s_1^* and retail price p_1^* for product 1 are decreasing in c_1 but increasing in c_2; (ii) the equilibrium service level s_2^* and retail price p_2^* for product 2 are decreasing in c_2 but increasing in c_1.*

Proof of Proposition 4. From Equations (9) and (10), we get the following derivatives:

$$\begin{aligned}
\frac{\partial s_1^*}{\partial c_1} &= \frac{\partial s_2^*}{\partial c_2} = \frac{-\Lambda_p(\gamma_p\gamma_s - 2\Lambda_p\Lambda_s)}{m(\gamma_p^2 - 4\Lambda_p^2)} \\
\frac{\partial s_1^*}{\partial c_2} &= \frac{\partial s_2^*}{\partial c_1} = \frac{-\Lambda_p(2\Lambda_p\gamma_s - \gamma_p\Lambda_s)}{m(\gamma_p^2 - 4\Lambda_p^2)}
\end{aligned}$$

It is easily proved that $\frac{\partial s_1^*}{\partial c_1} = \frac{\partial s_2^*}{\partial c_2} < 0$ and $\frac{\partial s_1^*}{\partial c_2} = \frac{\partial s_2^*}{\partial c_1} > 0$, if $\gamma_s > \beta_s$. Taking the first derivatives of p_1^* with respect to c_1 and c_2 yields

$$\begin{aligned}
\frac{\partial p_1^*}{\partial c_1} &= \frac{\partial p_1^*}{\partial s_1^*}\frac{\partial s_1^*}{\partial c_1} + \frac{\partial p_1^*}{\partial s_2^*}\frac{\partial s_2^*}{\partial c_1} \\
&= \frac{2\Lambda_p\Lambda_s - \gamma_s\gamma_p}{4\Lambda_p^2 - \gamma_p^2}\frac{\partial s_1^*}{\partial c_1} + \frac{\gamma_p\Lambda_s - 2\Lambda_p\gamma_s}{4\Lambda_p^2 - \gamma_p^2}\frac{\partial s_2^*}{\partial c_1}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial p_1^*}{\partial c_2} &= \frac{\partial p_1^*}{\partial s_1^*}\frac{\partial s_1^*}{\partial c_2} + \frac{\partial p_1^*}{\partial s_2^*}\frac{\partial s_2^*}{\partial c_2} \\
&= \frac{2\Lambda_p\Lambda_s - \gamma_s\gamma_p}{4\Lambda_p^2 - \gamma_p^2}\frac{\partial s_1^*}{\partial c_2} + \frac{\gamma_p\Lambda_s - 2\Lambda_p\gamma_s}{4\Lambda_p^2 - \gamma_p^2}\frac{\partial s_2^*}{\partial c_2}
\end{aligned}$$

Because $\frac{2\Lambda_p\Lambda_s - \gamma_s\gamma_p}{4\Lambda_p^2 - \gamma_p^2} > 0$, $\frac{\gamma_p\Lambda_s - 2\Lambda_p\gamma_s}{4\Lambda_p^2 - \gamma_p^2} < 0$, $\frac{\partial s_1^*}{\partial c_1} = \frac{\partial s_2^*}{\partial c_2} < 0$ and $\frac{\partial s_2^*}{\partial c_1} = \frac{\partial s_1^*}{\partial c_2} > 0$, we could get $\frac{\partial p_1^*}{\partial c_1} < 0$ and $\frac{\partial p_1^*}{\partial c_2} > 0$. Therefore, we note that the equilibrium service level s_1^* and p_1^* for product 1 is decreasing in c_1 but increasing in c_2. The proof of (ii) could be completed in a similar way. □

Proposition 4 tells us that a greater unit production cost of product i leads to lower equilibrium decisions in service level s_i and sales price p_i, however, a greater unit production cost of product $j (j \neq i)$ leads to greater equilibrium decisions in service level s_i and sales price p_i. It is observed in the proof of Proposition 4 that production cost i has a direct effect on equilibrium service levels s_i and s_j, while it exerts an indirect effect on equilibrium prices p_i and p_j. If a higher production cost of product i is set, the manufacturer will offer less service for product i to save cost, and decreasing equilibrium service level s_i lowers product i's selling price and increases product j's $(j \neq i)$ selling price.

Proposition 5. *(i) If $w_2 - c_2 > w_1 - c_1$, the equilibrium service level s_1^* and retail price p_1^* for product 1 are decreasing in γ_s; the equilibrium service level s_2^* and retail price p_2^* for product 2 are increasing in γ_s; (ii) If $w_2 - c_2 < w_1 - c_1$, the equilibrium service level s_1^* and retail price p_1^* for product 1 are increasing in γ_s; the equilibrium service level s_2^* and retail price p_2^* for product 2 are decreasing in γ_s.*

Proof of Proposition 5. Taking the first derivatives of s_1^* and s_2^* with respect to γ_s, if $w_2 - c_2 > w_1 - c_1$, we get

$$\begin{aligned}\frac{\partial s_1^*}{\partial \gamma_s} &= \frac{\Lambda_p[(w_1 - c_1)(\gamma_p - 2\Lambda_p) + (w_2 - c_2)(2\Lambda_p - \gamma_p)]}{m(\gamma_p^2 - 4\Lambda_p^2)} \\ &= \frac{\Lambda_p(2\Lambda_p - \gamma_p)[(w_2 - c_2) - (w_1 - c_1)]}{m(\gamma_p^2 - 4\Lambda_p^2)} < 0\end{aligned}$$

$$\frac{\partial s_2^*}{\partial \gamma_s} = -\frac{\Lambda_p(2\Lambda_p - \gamma_p)[(w_2 - c_2) - (w_1 - c_1)]}{m(\gamma_p^2 - 4\Lambda_p^2)} > 0$$

The first derivatives of p_1^* and p_2^* with respect to γ_s are as follows:

$$\begin{aligned}\frac{\partial p_1^*}{\partial \gamma_s} &= \frac{2\Lambda_p(s_1^* + \Lambda_s\frac{\partial s_1^*}{\partial \gamma_s} - s_2^* - \gamma_s\frac{\partial s_2^*}{\partial \gamma_s}) + \gamma_p(s_2^* + \Lambda_s\frac{\partial s_2^*}{\partial \gamma_s} - s_1^* - \gamma_s\frac{\partial s_1^*}{\partial \gamma_s})}{4\Lambda_p^2 - \gamma_p^2} \\ &= \frac{(2\Lambda_p - \gamma_p)(s_1^* - s_2^*) + (2\Lambda_p\Lambda_s - \gamma_p\gamma_s)\frac{\partial s_1^*}{\partial \gamma_s} + (\gamma_p\Lambda_s - 2\Lambda_p\gamma_s)\frac{\partial s_2^*}{\partial \gamma_s}}{4\Lambda_p^2 - \gamma_p^2}\end{aligned}$$

$$\begin{aligned}\frac{\partial p_2^*}{\partial \gamma_s} &= \frac{2\Lambda_p(s_2^* + \Lambda_s\frac{\partial s_2^*}{\partial \gamma_s} - s_1^* - \gamma_s\frac{\partial s_1^*}{\partial \gamma_s}) + \gamma_p(s_1^* + \Lambda_s\frac{\partial s_1^*}{\partial \gamma_s} - s_2^* - \gamma_s\frac{\partial s_2^*}{\partial \gamma_s})}{4\Lambda_p^2 - \gamma_p^2} \\ &= \frac{(2\Lambda_p - \gamma_p)(s_2^* - s_1^*) + (2\Lambda_p\Lambda_s - \gamma_p\gamma_s)\frac{\partial s_2^*}{\partial \gamma_s} + (\gamma_p\Lambda_s - 2\Lambda_p\gamma_s)\frac{\partial s_1^*}{\partial \gamma_s}}{4\Lambda_p^2 - \gamma_p^2}\end{aligned}$$

We get $s_2^* - s_1^* > 0$ from Proposition 2 and $\frac{\partial s_1^*}{\partial \gamma_s} < 0$ and $\frac{\partial s_2^*}{\partial \gamma_s} < 0$ if $w_2 - c_2 > w_1 - c_1$. Besides, if we also impose $\gamma_s > \beta_s$, it is apparent that $\gamma_p\Lambda_s - 2\Lambda_p\gamma_s < 0$ and $2\Lambda_p\Lambda_s - \gamma_p\gamma_s > 0$. Hence, we easily obtain $\frac{\partial p_1^*}{\partial \gamma_s} < 0$ and $\frac{\partial p_2^*}{\partial \gamma_s} > 0$. The proof of (i) is completed; (ii) could be easily demonstrated in the same way. □

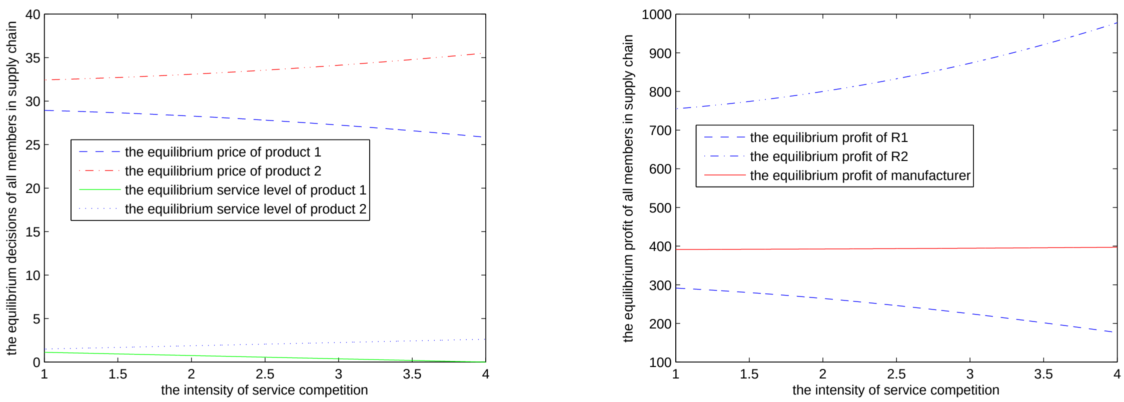
It is easily known that $w_2 - c_2 > w_1 - c_1$ signifies the manufacturer can gain more profit from unit product 2 than unit product 1. So when the two products compete with each other more fiercely in service, the manufacture would like to choose a higher service level s_2 to increase demand for product 2, yet a lower service level s_1 to decrease demand for product 1. As a result, the retailer $R1$ has no choice but to decrease sales price p_1 to boost demand for product 1 in order to compensate for the negative influence of decreasing s_1, and a higher s_2 ensures retailer $R2$ more scope to heighten the sales price p_2. When $w_2 - c_2 < w_1 - c_1$, the result is reversed.

4. Numerical Simulation

In this section, we resort to numerical approaches to analyze the chain members' equilibrium decision functions. The objective of the numerical analyses is twofold. One is to verify some important properties obtained in Section 3. The other is to numerically examine how the optimal policies change when the values of the problem parameters change. We mainly fix our eyes on the effects of the intensity of service and price competition and the service investment cost on chain members' equilibrium profits. Some concerning parameters are set as follows: $r_p = 1$, $\beta_p = 1/2$, $\gamma_s = 3$, $\beta_s = 2$, $m = 2$.

Figure 1a,b indicate how the equilibrium decisions and profits of all chain members change with respect to the intensity of service competition in the scenario $w_2 - c_2 > w_1 - c_1$ and $\Lambda_p(c_1 - c_2) < \alpha_2 - \alpha_1$. From Figure 1a, we know that the equilibrium price and service level of product 2 both increase in the intensity of service competition, while the equilibrium price and service level of product 1 both decrease in the intensity of service competition. Moreover, Figure 1a also tells us that the equilibrium price and service level of product 2 is larger than those of product 1, respectively. All these results are in line with Propositions 3, 4 and 5. In Figure 1b, we find that as the intensity of service competition increases, the retailer $R2$'s equilibrium profit increases, the retailer $R1$'s equilibrium profit decreases; however, the manufacturer's profit stays almost on the same level. This figure could be explained in the following way. Actually, when the intensity of service competition grows, the product's service level plays a more important role in aggrandizing itself market demand. It is also known that, when other parameter hold equal, the equilibrium service level of product 2 is greater than that of product 1. And as shown in Figure 1a, when the intensity of service competition increases, the service level of product 2 increases, while the service level of product 1 decreases. Consequently, the retailer $R2$ with a higher service level become more dominant in market than the retailer $R1$, as γ_s increases. Hence, the retailer $R2$'s equilibrium profit is increasing in the intensity of service competition, and the retailer $R1$'s equilibrium profit behaves in the opposite way. The manufacturer's more investment in product 2's service level is compensated by the declining investment in product 1's service level, so the manufacturer's equilibrium profit is not heavily dependent on the intensity of service competition.

Figure 1. $w_1 = 15, c_1 = 9, w_2 = 10, c_2 = 2, \alpha_1 = 30, \alpha_2 = 50.$

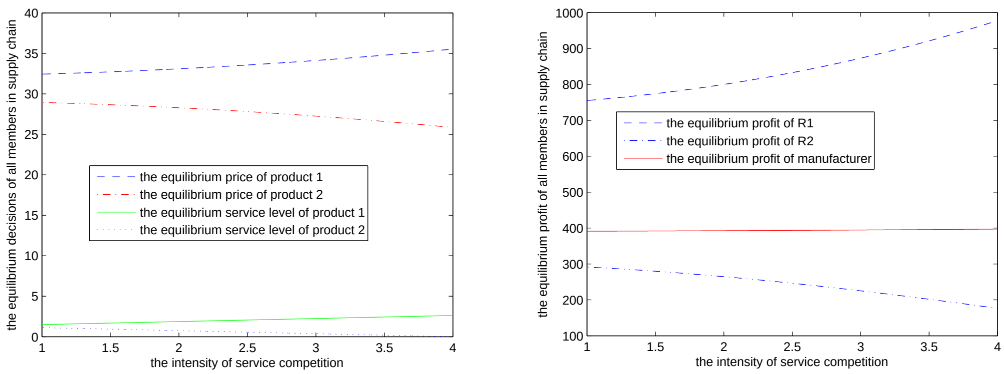


(**a**) (**b**)

Figure 2a,b also explore the varying of all chain members' equilibrium decisions and profits when the intensity of service competition γ_s changes under the condition $w_2 - c_2 < w_1 - c_1$ and $\Lambda_p(c_1 - c_2) >$

$\alpha_2 - \alpha_1$. From Figure 2a, we know that the equilibrium price and service level of product 1 both increase, while the equilibrium price and service level of product 2 both decrease, in the intensity of service competition. Besides, Figure 2a also tells us that the equilibrium price and service level of product 1 is larger than those of product 2, respectively. Figure 2b shows that when the intensity of service competition increases, the retailer $R1$'s equilibrium profit increases, the retailer $R2$'s equilibrium profit decreases, however, the manufacturer's profit perform minor change. Figure 3a–c all investigate the influence of the intensity of price competition on the chain members' equilibrium profit in the scenario $w_2 - c_2 > w_1 - c_1$ and $\Lambda_p(c_1 - c_2) < \alpha_2 - \alpha_1$. As seen from Figure 3a, the retailer $R1$'s equilibrium profit is always decreasing in the intensity of service competition. In addition, it is evident that fiercer price competition brings less profit to the retailer $R1$, when other parameters hold equal. That is because when price competition becomes more fierce, the retailer $R1$ will lower selling price to maintain an advantageous position in price competition. It is definitely helpful to boost the demand for product 1, however, the profit enhanced by the surge in demand is not enough to offset the loss caused by the declining selling price. In Figure 3b, the retailer $R2$'s equilibrium profit is always increasing in the intensity of service competition, and fiercer price competition declines retailer $R2$'s profit. Figure 3c depicts the manufacturer's equilibrium profit is increasing in the intensity of service competition. In addition, we also get that a larger γ_p augments the manufacturer's profit if other parameters are equal.

Figure 2. $w_1 = 10, c_1 = 2, w_2 = 15, c_2 = 9, \alpha_1 = 50, \alpha_2 = 30.$

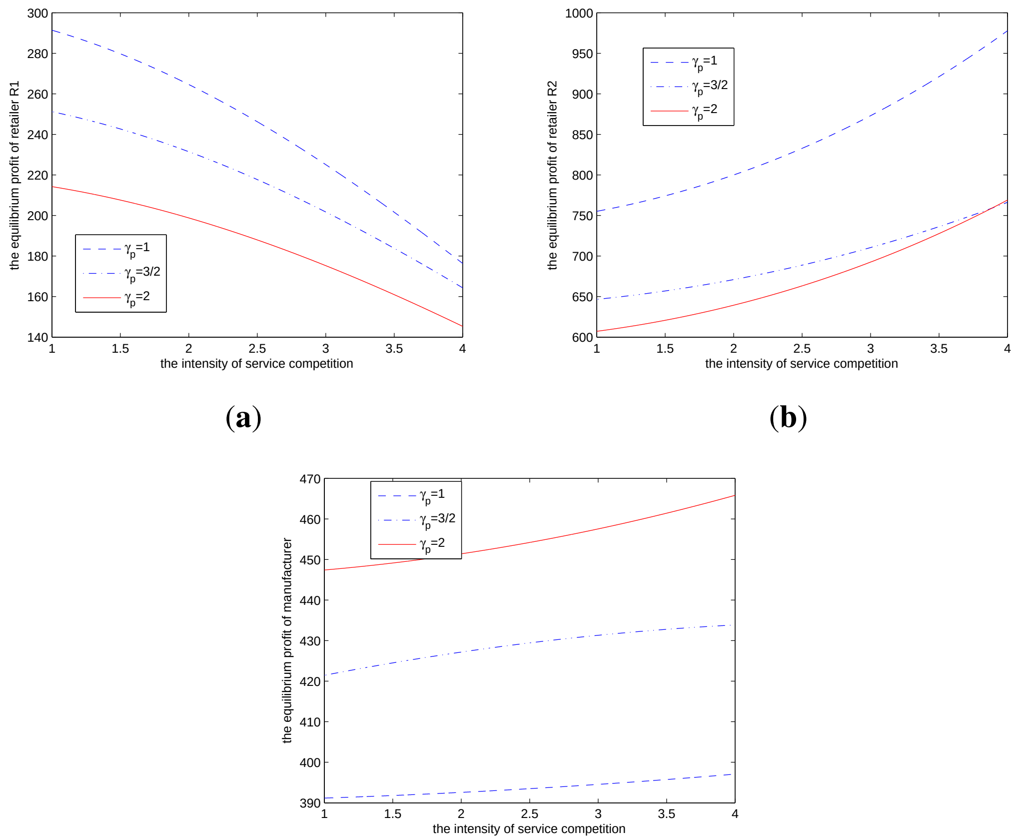


(a) **(b)**

Figure 4 validates what we observe in Figure 3a–c. Not only retailer $R1$'s profit but also the retailer $R2$'s profit is reduced due to the increase in the intensity of price competition γ_p, whereas the manufacturer's profit is increasing in γ_p. The effects of the service investment cost on the chain members' equilibrium profits are further investigated in Figures 5 and 6. Figure 5 is set in the scenario $w_2 - c_2 > w_1 - c_1$ and $\Lambda_p(c_1 - c_2) < \alpha_2 - \alpha_1$, while Figure 6 in the scenario $w_2 - c_2 < w_1 - c_1$ and $\Lambda_p(c_1 - c_2) > \alpha_2 - \alpha_1$. As shown in Figure 5, the retailer $R2$'s and the manufacturer's profits both decrease in the service investment cost, however, the retailer $R1$'s profit increases in the service investment cost. In summary, increasing m is unfavorable for retailer $R2$ and manufacturer, yet favorable to retailer $R1$. We have known from Proposition 4 that the equilibrium service level of product 2 is larger than that of product 1 under the condition $w_2 - c_2 > w_1 - c_1$. Consequently, the manufacturer is more likely to decrease s_2 to save the total service cost, which make the retailer $R2$'s profit shrink.

A decreasing s_2 leads the service level of product 1 to be relatively more attractive than before, hence the retailer $R2$'s achieve more profit as m increases. In Figure 6, the equilibrium profits of retailer $R1$ and the manufacturer both decrease, however, the equilibrium profit of retailer $R2$ increases in the service investment cost. It could be interpreted in a similar way.

Figure 3. $w_1 = 15, c_1 = 9, w_2 = 10, c_2 = 2, \alpha_1 = 30, \alpha_2 = 50.$



(a) **(b)**

(c)

Figure 4. $w_1 = 15, c_1 = 9, w_2 = 10, c_2 = 2, \alpha_1 = 30, \alpha_2 = 50.$

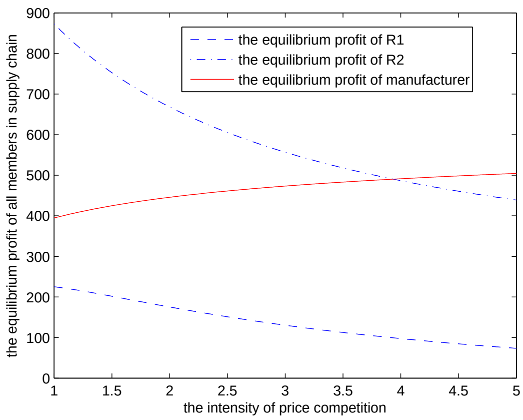


Figure 5. $w_1 = 15, c_1 = 9, w_2 = 10, c_2 = 2, \alpha_1 = 30, \alpha_2 = 50.$

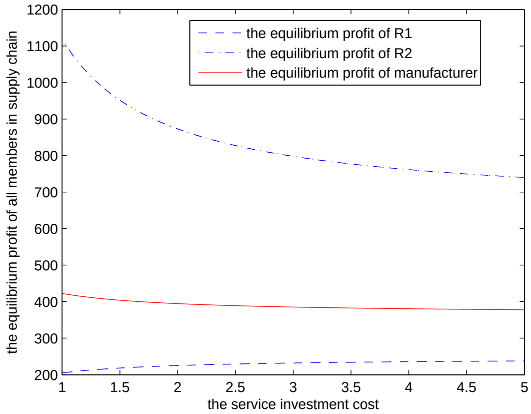
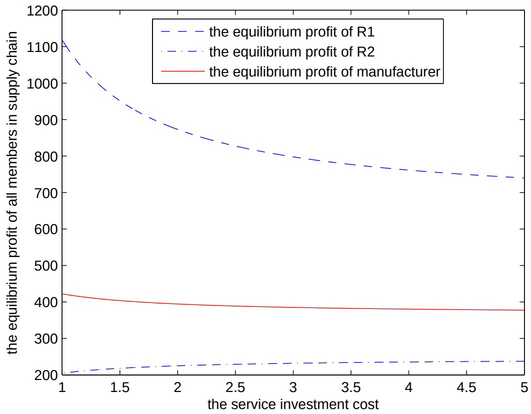


Figure 6. $w_1 = 10, c_1 = 2, w_2 = 15, c_2 = 9, \alpha_1 = 50, \alpha_2 = 30.$



5. Conclusions and Future Directions

The dynamics of chain members' decisions in the face of price and service competition have been one of the major concerns in the supply chain management. In this paper, we refer to a general demand function to describe the influence of the price and service competition on market demand in such a competitive environment. Moreover, a Stackelberg model structure is proposed between the two retailers and manufacturer in a two-echelon supply chain. We employ a backward approach in order to obtain the chain members' best response functions, and to determine the optimal service levels and selling prices of the two products, respectively. Furthermore, we explore the structure of the equilibrium decisions and study the impacts of some important parameters on the optimality. Lastly, numerical examples are also provided to verify the theoretical results and perform sensitivity analysis to derive managerial insights for practical applications. It is desirable that some useful conclusions are derived. We know the equilibrium service level and sales price of one product are both decreasing in its production cost but increasing in its rival product's production cost. It is also presented that when manufacturer can gain more profit from unit product i than unit product j ($i \neq j$), he would like to raise service level for product i and decrease service level for product j and retailer i would like to raise retail price of product i and retailer j decreases that of product j, if the two products compete with each other more fiercely in service. Furthermore, we also prove the equilibrium service level of product i is greater than that of product j on this condition. What is more, if one more restriction is imposed, we could make sense of the relation between the equilibrium retail prices of the two competitive products.

However, our research leaves several unanswered questions for future research. The demand function is assumed to be deterministic throughout the paper, but in the real world the market demand often occurs stochastically. Formulating a demand function incorporating stochastic variables will be an interesting extension of the paper. We also impose some restrictions on concerning parameters when analyzing the equilibrium decisions, so it is meaningful to explore how the equilibrium decisions change if the restrictions are relieved. Another limitation of the model is that we are unable to make a comparison between the two retailers' equilibrium profits because of the intricate form of the optimal decisions. So how to simplify the general model would be another important direction.

Acknowledgments

This research is supported by the National Natural Science Foundation of China (No. 71002106 and No. 71302112), the Independent Innovation Foundation of Tianjin University (No. 2014XRG-0100), the Humanities and Social Sciences Project of Shandong Provincial Education Department (No. J13WG19 and No. J14WG09).

Author Contributions

Xiaonan Han wrote the manuscript and participated in all phases. Xiaochen Sun conceived and guided the whole work. Yancong Zhou helped perform the analysis with constructive discussions. All authors have read and approved the final manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

References

1. Xiao, T.J.; Yang, D.Q. Price and service competition of supply chains with risk-averse retailers under demand uncertainty. *Int. J. Prod. Econ.* **2008**, *114*, 187–200.
2. Choi, S.C. Price competition in a channel structure with a common retailer. *Mark. Sci.* **1991**, *10*, 271–296.
3. Yan, R.L.; Ghose, S.J.; Bhatnagar, A. Cooperative advertising in a dual channel supply chain. *Int. J. Electron. Mark. Retail.* **2006**, *1*, 99–114.
4. Liu, S.K.; Xu, Z.S. Stackelberg game models between two competitive retailers in fuzzy decision environment. *Fuzzy Optim. Decis.* **2014**, *13*, 33–48.
5. Ertek, G.; Griffin, P.M. Supplier- and buyer-driven channels in a two-stage supply chain. *IIE Trans.* **2003**, *34*, 691–700.
6. Yang, S.L.; Zhou, Y.W. Two-echelon supply chain models: Considering duopolistic retailers' different competitive behaviors. *Int. J. Prod. Econ.* **2006**, *103*, 104–116.

7. Hsieh, C.-C.; Wu, C.-H. Coordinated decisions for substitutable products in a common retailer supply chain. *Eur. J. Oper. Res.* **2009**, *196*, 273–288.
8. Ba, S.; Stallaert, J.; Zhang, Z. Oligopolistic price competition and adverse price effect in online retailing markets. *Decis. Support Syst.* **2008**, *45*, 858–869.
9. Lyer, G. Coordinating channels under price and nonprice competition. *Mark. Sci.* **1998**, *17*, 338–355.
10. Tsay, A.A.; Agrawal, N. Channel dynamics under price and service competition. *Manuf. Serv. Oper. Manag.* **2000**, *2*, 372–391.
11. Xia, Y.; Gilbert, S.M. Strategic interactions between channel structure and demand enhancing services. *Eur. J. Oper. Res.* **2007**, *181*, 252–265.
12. Wu, C.-H. Price and service competition between new and remanufactured products in a two-echelon supply chain. *Int. J. Prod. Econ.* **2012**, *140*, 496–507.
13. Zhao, J.; Liu, W.Y.; Wei, J. Competition under manufacturer service and price in fuzzy environments. *Knowl.-Based Syst.* **2013**, *50*, 121–133.