

Article

Kron Reduction Based on Node Ordering Optimization for Distribution Network Dispatching with Flexible Loads

Huihui Song¹ , Linkun Han², Yichen Wang³, Weifeng Wen¹ and Yanbin Qu^{1,*} 

¹ School of New Energy, Harbin Institute of Technology, Weihai 264209, China; songhh@hitwh.edu.cn (H.S.); wwfweng2022@163.com (W.W.)

² State Grid Tai'an Electric Power Company, Tai'an 271000, China; shanlin13@163.com

³ School of Electrical and Electronic Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore; wang1626@e.ntu.edu.sg

* Correspondence: quyanbin@hit.edu.cn

Abstract: Kron reduction is a general tool of network simplification for flow calculation. With a growing number of flexible loads appearing in distribution networks, traditional Kron reduction cannot be widely used in control and scheduling due to the elimination of controllable and variable load buses. Therefore, this paper proposes an improved Kron reduction based on node ordering optimization whose principles guarantee that all the boundary nodes are retained eventually after eliminating the first row and the first column in every step according to the order, thereby making it possible to take full advantage of their potential to meet different requirements in power system calculation and dispatching. The proposed method is verified via simulation models of IEEE 5-bus and 30-bus systems through illustrating the dynamic consistency of the output active power of the generator nodes and the power flow data of preserved nodes before and after reduction.

Keywords: Kron reduction; node ordering optimization; flexible loads; distribution network dispatching; graph theory



Citation: Song, H.; Han, L.; Wang, Y.; Wen, W.; Qu, Y. Kron Reduction Based on Node Ordering Optimization for Distribution Network Dispatching with Flexible Loads. *Energies* **2022**, *15*, 2964. <https://doi.org/10.3390/en15082964>

Academic Editor: Pavlos S. Georgilakis

Received: 22 March 2022

Accepted: 15 April 2022

Published: 18 April 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Nowadays, smart grids lead the development of power systems, and plenty of distributed energy resources (DERs) [1], including electric vehicles (EV) [2] and other flexible loads [3], especially air-conditioning [4], have become significant components in distribution networks, as shown in Figure 1. Different from traditional loads, these special loads can achieve a bidirectional energy exchange and elastic demand according to the running conditions [5], which means that they have immense potential to maintain the power balance [6]. Therefore, to make full use of their capacity in power system scheduling, EVs and other flexible loads cannot be equivalent to line impedance simply reduced in traditional Kron reduction.

Network simplification, for effectiveness in reducing the time and enhancing the accuracy of calculations, provides a wonderful foundation for power network scheduling [7]. Commonly used static network reduction techniques include Ward reduction, Kron reduction, Dimo's method, and Zhukov's method [8]. As a classical method for power network partition, block computing was first proposed by Kron dating back to the 1950s. This approach aims to reduce networks by removing unimportant nodes and edges without changing the dynamic characteristics of the system [9], and it has gradually become a profound tool in simplifying electrical networks. Traditional Kron reduction is available for the power flow calculation of linear electrical networks [10] and became quite sophisticated around the 1990s.

In 2012, Dorfler et al. proposed the Kron reduction of graphs and depicted its application to power networks [11]. They carried out a comprehensive analysis in topology, algebra, resistance, spectrum, and sensitivity and obtained a simplified network with only

boundary nodes. Benefitting from this analysis, researchers utilized and improved Kron reduction to make it suitable for different applications. Considering that Kron reduction was used to simplify the analysis of multi-machine power systems under certain steady state assumptions, ref. [12] showed how to perform Kron reduction for a class of electrical networks without steady state assumptions so that the reduced model can be used to analyze the transient state behavior of these electrical networks. In terms of nonlinear circuits, Kron reduction was used in [13] as a procedure that isolates the interactions of the nonlinear circuits, which enabled sufficient conditions derived for global asymptotic synchronization in a system of identical nonlinear electrical circuits coupled through linear time-invariant electrical networks. Moreover, ref. [14] introduced the projected incidence matrix in the linear and the nonlinear differential algebraic model of the network and thus provided a complementary approach to Kron reduction, which is able to cope with constant power loads and nonlinear power flow equations. Afterwards, since existing approaches to instantaneous Kron reduction can fail in preserving the structure of transfer functions representing power lines, ref. [15] proposed two approximate Kron reduction algorithms capable of representing exactly the asymptotic behavior of electric signals. In addition, some improved Kron reduction methods are applied to microgrid reconfiguration [16], distribution systems modeling [17,18], and network dispatching. Ref. [19] was concerned with a multi-area available transfer capacity (ATC) evaluation method based on Kron reduction. Through four steps including system decomposition, region equivalence, data exchange, and topology correction, the computational accuracy is improved while ensuring the timeliness of online calculation of ATC. An iterative Kron reduction was then applied in [20] to eliminate most of the buses that are not connected to any inter-area line, which leads to the same investment decisions in a transmission expansion planning (TEP) context as if the whole original network were considered. However, most of the current literature only concerns the dynamics consistency or control properties of generators (usually regarded as PV nodes), while the majority of load buses will be eliminated as PQ nodes and taken as the line impedance between generator nodes. To settle this problem, a modified Kron reduction for distribution networks with node ordering optimization is proposed and verified in this paper. It adopts a modified ordering strategy that numbers the internal nodes to be reduced before the boundary nodes in the sequence, ‘slims down’ the whole admittance matrix along the diagonal layer by layer, and thereby the nodes of EVs and other flexible loads can be retained while eliminating uncontrollable nodes. Consequently, the potential of EVs and other flexible loads can also be retained in the process of reduction through the proposed adaptive method, which will add tremendous value in power grid dispatching. This improvement makes Kron reduction more suitable for power system control and scheduling, as well as extending its application range.

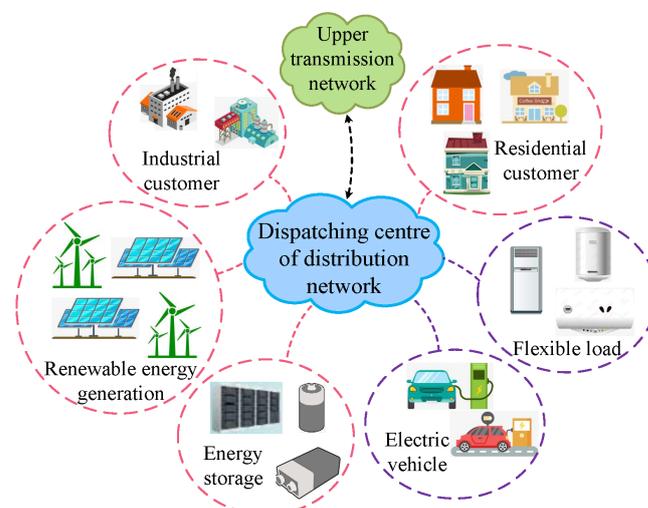


Figure 1. Schematic of distribution network composition.

The contributions of this paper can be summarized as follows: the presentation of the improved Kron reduction and node ordering optimization are the main innovation points. Through this method, all the load buses can be preserved or eliminated selectively to maximize the advantage of flexible loads while ensuring conciseness. Moreover, the node ordering pattern guarantees the simplicity of the reduction process. Furthermore, the proposed method is applied to IEEE 5-bus and 30-bus test systems to prove its validity. The simulation results can fully demonstrate the equivalency and dynamic characteristic consistency between the model before and after the reduction.

The rest of the paper is organized as below: the traditional Kron reduction and graph theory are formulated in Section 2. Detailed explanations of the proposed Kron reduction method are provided in Section 3. The simulation results and conclusions are provided in Sections 4 and 5, respectively.

2. Traditional Kron Reduction and Graph Theory

2.1. Traditional Kron Reduction

Before the reduction, it is necessary to divide all nodes of a power network into two kinds—internal nodes and boundary nodes—and let i and b denote the number of these two kinds of nodes, respectively. The internal nodes are eliminated during the reduction and the boundary nodes retained. In traditional Kron reduction, the internal nodes are those whose current injections are zero (PQ nodes) and the boundary nodes are nodes with generators (PV and balance nodes) in general. Traditional stability analysis focuses on the variation in rotor angle with time rather than the variation in the voltage phase angle of the whole system node. Therefore, it is feasible to only retain the nodes with generators, which keeps the dynamic stability and reduces the dimension of the power network and the complexity of analysis and calculation. Thus, the current balance equation $\mathbf{I} = \mathbf{YU}$ can be written as

$$\begin{pmatrix} 0 \\ I_b \end{pmatrix} = \begin{pmatrix} Y_{ii} & Y_{ib} \\ Y_{bi} & Y_{bb} \end{pmatrix} \begin{pmatrix} U_i \\ E_b \end{pmatrix} \quad (1)$$

where $Y_{ib} = Y_{bi}^T$. Eliminate U_i as

$$I_b = (Y_{bb} - Y_{bi}Y_{ii}^{-1}Y_{ib})E_b = Y_r E_b \quad (2)$$

where

$$Y_r = Y_{bb} - Y_{bi}Y_{ii}^{-1}Y_{ib} \quad (3)$$

is the new node admittance matrix after Kron reduction, and it is known as the Schur complement of the node admittance matrix. Actually, during Kron reduction, the Schur complement of the node admittance matrix is repeatedly used to eliminate the internal nodes.

2.2. Graph Theory Concepts

As the power network is a net and we are interested in the connection between nodes and power transmission, it is clear and intuitive to associate the simplification process of a power network with the equivalent change in the connection graph based on the study of Florian [11]. Here, its relative concepts are given.

2.2.1. Power Network and Undirected Graph

Consider an undirected weighted and connected graph $D=(V(D), C(D))$ and its vertex set $V(D)$ and edge set $C(D)$. This graph corresponds to a network, while its vertexes and edges correspond to nodes and branches of the network, respectively. $c = (v_i, v_j)$ of $C(D)$ denotes the edge that links vertex i and j . The relation between the power network and undirected graph can be described as follows:

1. Every vertex of $V(D)$ is mapping to both the element of \mathbf{I} and element of \mathbf{U} . Then, the relation can be written as $v_i \iff I_i \iff U_i$;
2. A_{ij} of the adjacency matrix \mathbf{A} is the weight of the arc $c = (v_i, v_j)$, where $A_{ij} = A_{ji}$ because graph D is an undirected graph. \mathbf{Y} can be acquired from \mathbf{A} as $Y_{ii} = A_{ii} + \sum_{j=1, j \neq i}^n A_{ij}$ and $Y_{ij} = -A_{ij}$;
3. $I_i \in \mathbf{I}$ satisfies $I_i = \sum_{j=1}^n Y_{ij}U_j$, ($U_j \in \mathbf{U}, Y_{ij} \in \mathbf{Y}$). Non-zero element Y_{ij} denotes that the arc c_{ij} exists, and zero element Y_{ij} denotes that v_i and v_j are not connected directly.

The relation between the coefficient matrix and undirected graph can be illustrated by Figure 2; it is not difficult to conclude that matrix calculation is related to graph behavior. Then, by adopting the power system network equation with the admittance matrix, the calculation of the admittance matrix can be shown as the change of graph.

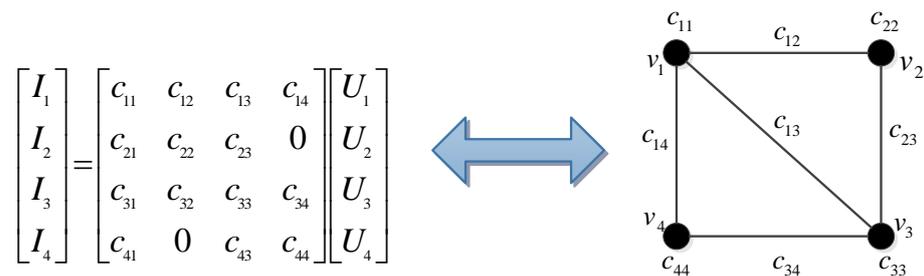


Figure 2. The relation between coefficient matrix and undirected graph.

2.2.2. Kron Reduction and Contraction of Graph

Kron reduction is closely connected to the contraction of the graph and the process of reducing nodes is given here:

1. Eliminate the vertex v_k ;
2. For any two vertexes v_i and v_j that both connect to v_k originally, create a new edge $c = (v_i, v_j)$ whose weight Y_{ij} can be extracted from \mathbf{Y}_r calculated according to Equation (3);
3. For the arc that did not change during the elimination, renew its weight also through Equation (3). For instance, we eliminate v_1 of the network shown in Figure 2 and its corresponding contracted graph is shown in Figure 3.

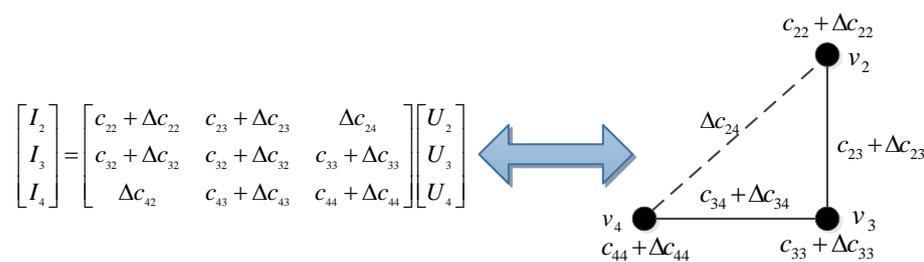


Figure 3. The relation between coefficient matrix and contracted graph.

3. Kron Reduction with Node Ordering Optimization

In this section, an improved Kron reduction with node ordering optimization is proposed and its detailed reduction process is provided. To preserve important load buses of the power system, a node by node iterative elimination method is adopted and some new criteria for node classifying and ordering are put out accordingly.

3.1. Model Establishment and Node Classification

As described in Section 2, the current balance equation is adopted to describe the power system model. Therefore, the properties of node admittance matrix \mathbf{Y} in the current balance equation are briefly introduced first:

- \mathbf{Y} is a square matrix and the number of rows and columns is equal to the number of nodes other than the reference node;
- \mathbf{Y} is a sparse matrix. For off-diagonal elements in each row, the number of non-zero elements is equal to the number of ungrounded branches of the node corresponding to the row;
- For each row, the diagonal element is equal to the sum of the admittance of the branches connected to the node;
- The off-diagonal element Y_{ij} is equal to the opposite of admittance of the branch between node i and node j and thus \mathbf{Y} is symmetric;
- It is easy to find that \mathbf{Y} can be obtained from the adjacency matrix \mathbf{A} directly.

The nodes connected to the ground are widespread in actual power systems, whose self-loops will affect the process of the reduction and the form of the reduced matrix. Therefore, we introduce a hypothetical additional grounded node with the number $n + 1$ and the augmented admittance matrix $\hat{\mathbf{Y}} \in \mathbb{C}^{(n+1) \times (n+1)}$ is constructed as

$$\hat{\mathbf{Y}} = \begin{pmatrix} \mathbf{Y} & -diag(\{A_{ii}\}_{i=1}^n)\mathbf{1}_n \\ -\mathbf{1}_n^T diag(\{A_{ii}\}_{i=1}^n) & \sum_{i=1}^n A_{ii} \end{pmatrix} \tag{4}$$

where $\mathbf{A} \in \mathbb{C}^{n \times n}$ is the adjacency matrix of graph D , $\{A_{ii}\}_{i=1}^n$ is a one-dimensional array and $diag(\{A_{ii}\}_{i=1}^n) \in \mathbb{C}^{n \times n}$ is the corresponding diagonal matrix. $\mathbf{1}_n$ is the one-dimensional identity matrix. The changes in graph and admittance matrix after introducing the additional grounded node are shown in Figure 4.

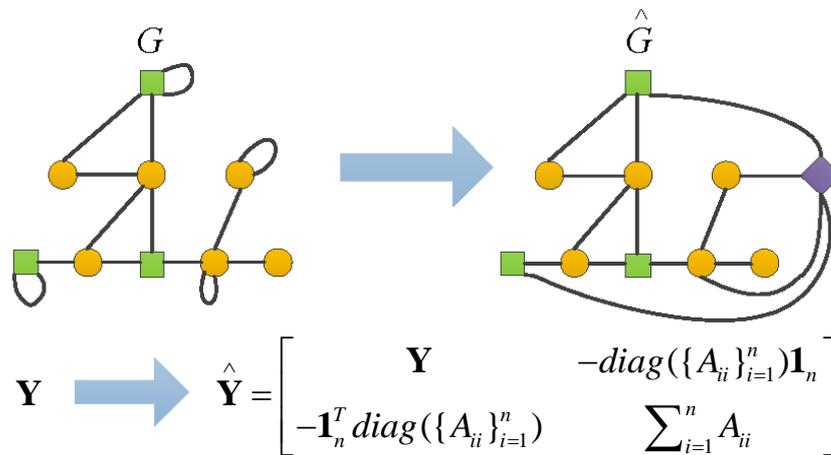


Figure 4. Changes after introducing additional grounded node (diamond icon) to a network with 4 boundary nodes (square icon), 6 internal icons (circle icon).

Unlike the traditional Kron reduction discussed in Section 2, the proposed Kron reduction in this paper allows users to reclassify nodes according to their requirements, but there are still some rules to follow:

1. Regard the nodes with generators as boundary nodes;
2. Regard the additional grounded node as a boundary node during simplification, but it will be eliminated to restore the graph after the final step of Kron reduction;
3. When considering the influence of controllable nodes with electric vehicles and other flexible loads, these non-generator nodes can also be regarded as boundary nodes.

For the first principle, traditional transient stability analysis concerns the change in rotor phase angle over time but not the phase angle of all nodes during the transient process; as a result, regarding nodes with generators as boundary nodes is appropriate. As for the second principle, if an additional grounded node is eliminated as an internal node, a self-loop will appear once again in the corresponding graph. If this is the middle step of reduction, an additional grounded node is still needed to add to the network in the next step of reduction, which is obviously unnecessary. For the last principle, regarding

the concerned nodes as boundary nodes can easily help us to retain and acquire relative information of these nodes.

3.2. Node Ordering Optimization

A proper node order is beneficial to the reduction process as we may obtain a sparse admittance matrix, while nodes may be inappropriately numbered, which is difficult to cope with for programming. Therefore, based on the traditional ordering approaches, a modified ordering strategy is recommended for the proposed Kron reduction. Its principles are as below:

1. Number the internal nodes before the boundary nodes in the sequence;
2. Introduce the degree of node in graph theory to the power network. Define $d(i)$ as the degree of node i equal to the number of branches connected to node i in the undirected weighted and connected graph. Number the internal nodes by descending order of $d(i)$;
3. Based on the above two principles, try to number the adjacent nodes with closer numbers.

The explanations are given here. Principle (1) guarantees that all the boundary nodes are retained eventually, because we will eliminate the first row and the first column in every step of elimination according to the order. The aim of principle (2) is to reduce the number of non-zero elements added to the admittance matrix during elimination. Finally, principle (3) can guarantee that non-zero elements are arranged as closely as possible to the diagonal but not scattered, thereby simplifying the programming in practical application. After reordering nodes according to the above principles, the initial admittance matrix will become a diagonally dominant sparse matrix with most non-zero elements lying in diagonal or its two sides. Moreover, the reduction procedure will be much simpler and clearer when adopting this ordering technique.

3.3. The Process of the Proposed Kron Reduction

In this part, the “node by node” simplifying process of the proposed Kron reduction is detailed as follows:

1. The initial adjacency matrix \mathbf{A}^0 is obtained from the admittance of the known network in which diagonal element A_{ii}^0 is equal to the admittance between ground and node i and non-diagonal element A_{ij}^0 is equal to the branch admittance between node i and node j ;
2. Form the initial admittance matrix \mathbf{Y}^0 and augmented admittance matrix $\widehat{\mathbf{Y}}^0$. The diagonal elements of \mathbf{Y}^0 satisfy $Y_{ii}^0 = A_{ii}^0 + \sum_{j=1, j \neq i}^n A_{ij}^0$ and the non-diagonal elements satisfy $Y_{ij}^0 = -A_{ij}^0$. Then, \mathbf{Y}^0 can be transformed into $\widehat{\mathbf{Y}}^0$, as in Equation (4);
3. Eliminate each internal node through Equation (3) as the node order obtained from node ordering optimization. In the l^{th} reduction process, the reduced augmented admittance matrix $\widehat{\mathbf{Y}}^l$ is obtained from $\widehat{\mathbf{Y}}^{l-1}$. Then, we deduce reduced admittance matrix \mathbf{Y}^l from $\widehat{\mathbf{Y}}^l$ as Equation (4) and afterwards \mathbf{A}^l from \mathbf{Y}^l according to $A_{ij} = -Y_{ij}$ and $A_{ii} = Y_{ii} + \sum_{j=1, j \neq i}^n Y_{ij}$;
4. Repeat step 3 until the reduced network only includes nodes that need to be retained;
5. The $\widehat{\mathbf{Y}}^{re}$ is obtained in the last step. Then, we restore the final reduced admittance matrix \mathbf{Y}^{re} from $\widehat{\mathbf{Y}}^{re}$ according to Equation (4).

To demonstrate the process intuitively, the flow chart of reduction is shown in Figure 5.

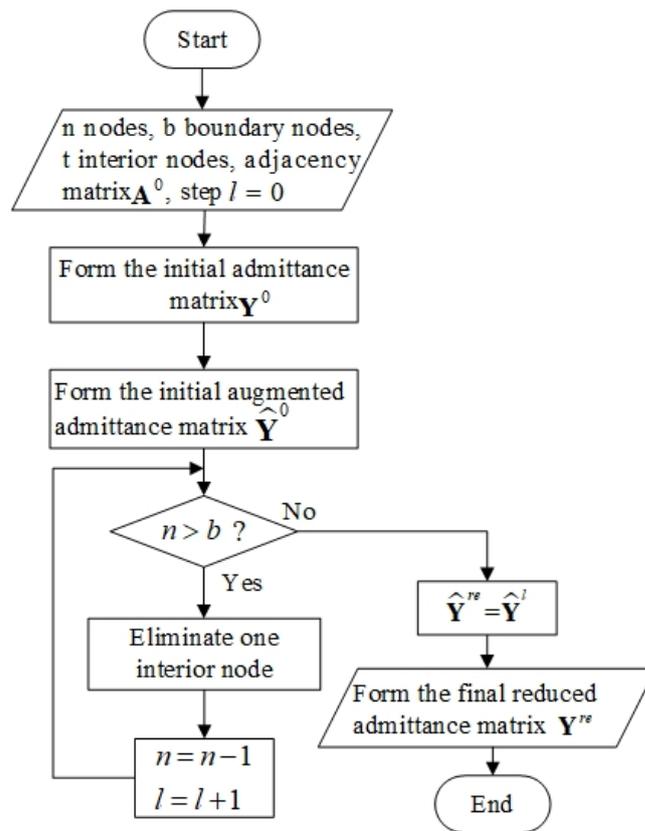


Figure 5. The flow chart of the proposed Kron reduction.

3.4. Application Instances of Proposed Kron Reduction

3.4.1. The Reduction of IEEE 5-Bus System

In the first instance, the IEEE 5-bus system with two generators and two nodes connected to the ground is simplified. To demonstrate the reduction procedure better, only two nodes with generators are regarded as boundary nodes. Number all nodes according to node ordering optimization. Then, the initial network can be expressed by graph as in Figure 6, and the admittance of every branch and the node number are also marked.

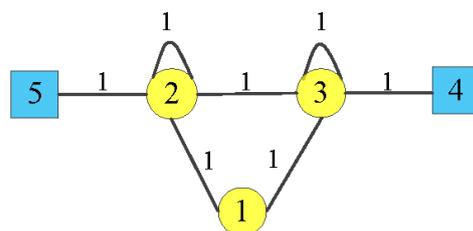


Figure 6. IEEE 5-bus system with 2 boundary nodes (square icon) and 3 interior nodes (circle icon).

After adopting the proposed Kron reduction and obtaining the admittance matrix and adjacency matrix of all steps, the graph topology corresponding to every adjacency matrix is as provided in Figure 7. We can conclude from the reduction process that the initial and final reduced networks are both symmetrical, though the symmetry is temporarily broken in the middle steps of reduction. To show the network changes after each reduction step more clearly and intuitively, we eliminate the additional grounded node and restore the graph in every step, which is unnecessary in practical application.

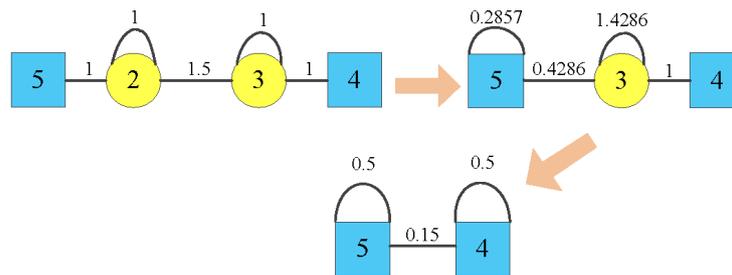


Figure 7. IEEE 5-bus system reduction process with proposed node ordering optimization.

Then, we reorder the nodes without the proposed node ordering optimization and repeat the reduction progress, whose details are shown in Figure 8. The results in Figure 8 can be analyzed from two aspects: on the one hand, after reordering the nodes, the middle steps of reduction are different but the eventual result is the same. It means that the elimination sequence will not influence the reduced network, which indicates the independence of internal nodes. On the other hand, by comparing networks after the first step, it is obvious that the added branches in Figure 8 are more than those in Figure 7, leading to more complicated calculation in the following reduction, especially in large-scale networks, which means that Kron reduction with node ordering optimization achieves a simpler calculation procedure than that without node ordering optimization. As a consequence, the superiority of node ordering optimization is evidenced.

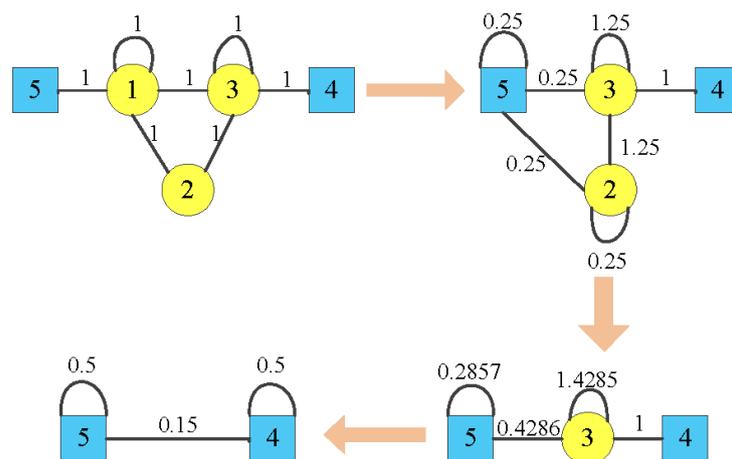


Figure 8. IEEE 5-bus system reduction process without proposed node ordering optimization.

3.4.2. The Reduction of IEEE 30-Bus System

For the IEEE 30-bus system shown in Figure 9, nodes 1, 2, 5, 8, 11 and 13 are with generators and nodes 3, 19 and 27 are with EVs or flexible loads, which should be eliminated as internal nodes in traditional Kron reduction, i.e., Kron reduction without node ordering optimization. These three controllable nodes, however, are supposed to be retained to make use of their potential in power system dispatching. Therefore, we regard these above 9 nodes as boundary nodes and reorder the whole system according to the principles of node ordering optimization. Through the proposed Kron reduction adopting the standard node data, internal nodes are eliminated “node by node”. The order of nodes and the simplified topology are shown in Figure 10. The line impedance of each branch for the reduced network is shown in Table 1.

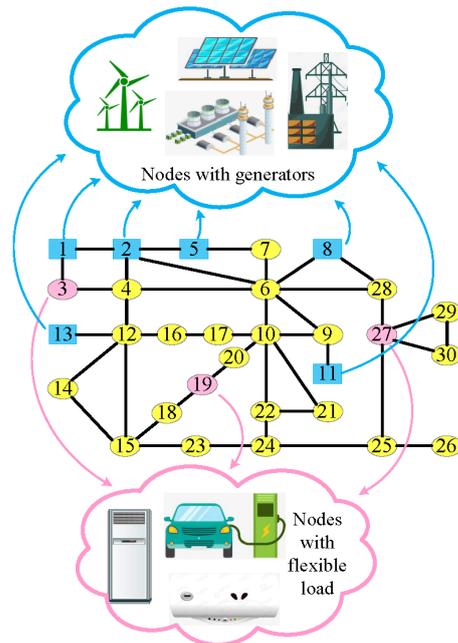
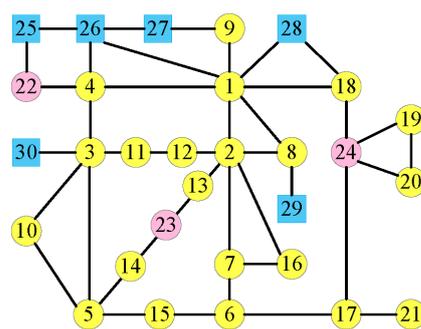
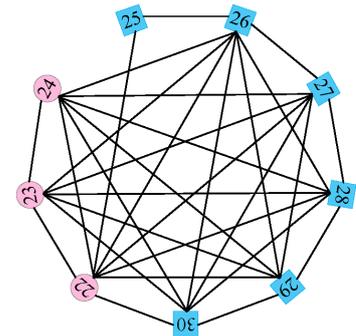


Figure 9. IEEE 30-bus system with EVs or flexible loads.



(a) Topology of the IEEE 30-bus system



(b) Topology of the reduced IEEE 30-bus system

Figure 10. IEEE 30-bus system before and after reduction. (a) Description of the order of nodes of the IEEE 30-bus system. (b) Description of simplified topology of the IEEE 30-bus system.

Table 1. Line impedance of the reduced IEEE 30-bus network.

Initial Node	End Node	Line Resistance (Ω)	Line Reactance (Ω)	Initial Node	End Node	Line Resistance (Ω)	Line Reactance (Ω)
22	23	6.753	0.460	24	27	16.829	1.508
22	24	13.443	0.608	24	28	4.660	0.178
22	25	4.520	0.049	24	29	5.120	1.180
22	26	9.035	0.065	24	30	12.294	1.039
22	27	10.798	0.281	25	26	1.920	0.015
22	28	6.574	0.052	26	27	4.473	0.046
22	29	-7.132	0.623	26	28	8.786	0.070
22	30	-8.188	0.257	26	29	-15.294	0.917
23	24	33.966	0.538	26	30	-24.857	0.787
23	26	114.348	0.975	27	28	7.603	0.112
23	27	6.227	2.062	27	29	-10.635	1.555
23	28	5.088	0.376	27	30	-21.786	2.761
23	29	26.247	0.359	28	29	-7.995	0.278
23	30	28.769	0.211	28	30	-14.649	0.494
24	26	39.143	0.895	29	30	-17.538	1.057

4. Simulation Verification

In this section, the IEEE 5-bus and 30-bus system reduction models are established and simulated with the tools of Matlab/Simulink. The node parameters before and after the proposed Kron reduction are compared to verify the equivalency of the network before and after reduction, and to demonstrate the effectiveness of node ordering optimization further. In this paper, the power flow data at the retained nodes and output characteristics of generators are chosen to prove the equivalency and dynamic consistency before and after reduction.

4.1. Equivalence and Restoration of Power Network Parameters

In a power network, all nodes with synchronous generators and internal nodes may carry loads. In the reduction process, a load can be regarded as a constant impedance and its admittance can be derived from

$$y_{Li} = \frac{P_{Li} - jQ_{Li}}{|U_i|^2}, i \in n \tag{5}$$

Let m denote the number of retained nodes and $Y_{Ln_1} = \text{diag}(y_{Li}), n_1 = n - m$ denote the equivalent load admittance matrix of eliminated nodes, and then the final admittance matrix of the network can be written in the form of

$$Y = \begin{pmatrix} Y_{mm} & Y_{mn_1} \\ Y_{n_1m} & Y_{n_1n_1} + Y_{Ln_1} \end{pmatrix} \tag{6}$$

where $Y_{mm} \in \mathbb{C}^{m \times m}$ is the boundary node matrix, $Y_{n_1n_1} \in \mathbb{C}^{n_1 \times n_1}$ is the interior node matrix, $Y_{n_1m} \in \mathbb{C}^{n_1 \times m}$ is the mutual admittance matrix of boundary nodes and interior nodes. Correspondingly, for the equivalent admittance matrix of retained nodes obtained by Kron reduction, Equation (7) converts it to the equivalent load of the nodes.

$$P_{Li} - jQ_{Li} = y_{Li} * |U_i| \tag{7}$$

In this way, we establish complete correspondence between the equivalent load and the node-to-ground equivalent admittance of the interior nodes, as

$$y_{Li} \iff P_{Li} - jQ_{Li}, i \in n_1 \tag{8}$$

In the following simulation process, we take the equivalent load as the operation parameter.

4.2. Verification of Equivalency and Dynamic Consistency

Let us consider a simple IEEE 5-bus system with resistive loads first, whose topology and node order are shown in Figure 6, and the system parameters are detailed in Table 2.

Table 2. Parameters of IEEE 5-bus system in simulation.

Parameters	Symbol	Value
Generator rating power	P_e	5×10^4 W
Frequency	f	60 Hz
Load at node 1	P_{L_1}	1000 W
Load at node 1	P_{L_2}	1000 W
Load at node 1	P_{L_3}	1000 W
Impedance on each line	Z_l	$(0.1 + j0.25)\Omega$

We follow the procedure demonstrated in Figure 7, and the corresponding models of 5-node, 4-node and 2-node networks are established, respectively. As indicated in Table 2, the ratio of line reactance to total system resistance is quite small, so we treat

the system as a pure resistive network to simplify calculation. We change the excitation rated value of generators to alter their output and thereby achieve the power balance of the system. Figure 11 and Table 3 illustrate the dynamic consistency of the output active power of two generator nodes and the power flow data of preserved nodes in the process of simplification separately. It is obvious that the output active power of two generator nodes and the power flow data of preserved nodes in the process of simplification are almost consistent. However, we also notice that there still remain some small deviations in the reduced network owing to the untreated line reactance and the limited calculation precision in the process of simplification.

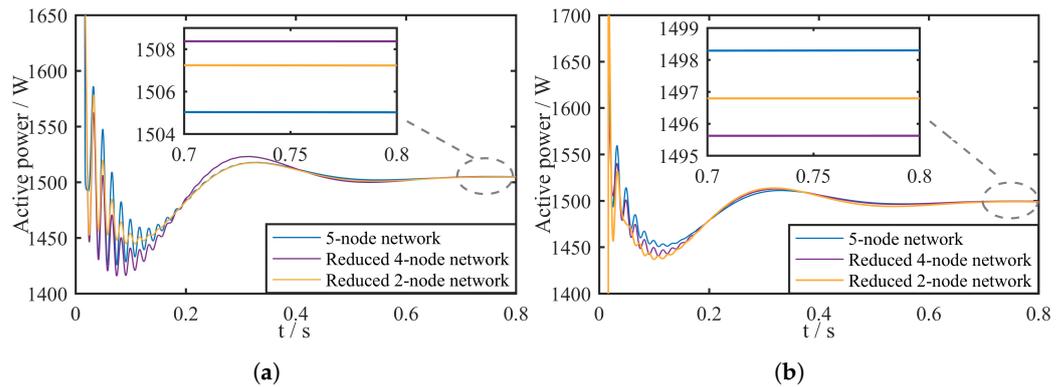


Figure 11. Output active power in the process of simplification of the generator node 4 in (a) and 5 in (b).

Table 3. The power flow data of preserved nodes after every reduction step.

Number of Nodes	Node Order	Voltage (U)		Generation		Load	
		Amplitude (V)	Phase Angle (deg)	P_G (W)	Q_G (Var)	P_L (W)	Q_L (Var)
5	1	374.68	-0.18	-	-	1000.00	0.00
	2	376.20	-0.13	-	-	1000.00	0.00
	3	376.20	-0.13	-	-	1000.00	0.00
	4	380.00	0.00	1505.02	7.87	-	-
	5	380.00	0.00	1498.29	0.00	-	-
4	2	376.59	-0.12	-	-	1000.00	0.00
	3	376.59	-0.12	-	-	1000.00	0.00
	4	380.00	0.00	1508.37	6.38	-	-
	5	379.62	0.00	1495.62	0.00	-	-
2	4	380.00	0.00	1507.22	7.31	-	-
	5	380.00	0.00	1496.97	0.00	-	-

Furthermore, to verify the effectiveness and correctness of the proposed method for larger-scale electrical networks, a model of an IEEE 30-bus system with standard parameters is also built and simulated. As analyzed in Section 3, the network only preserving the generator nodes is the final and simplest result of the proposed “node by node” reduction. If the equivalency and dynamic consistency of this simplest result are verified, the correctness of each simplification step can be proven at the same time, so only the simulation results of the original system and the reduced system only with generator nodes are shown here. Based on the results in Table 1, we further eliminate nodes 22 to 24. The final reduced admittance matrix and the corresponding line impedance are shown as Equation (9) and Table 4.

$$Y_{30}^{re} = \begin{pmatrix} 6.15 - 19.14i & -5.59 + 16.79i & -0.09 + 0.27i & -0.42 + 1.53i & 0.01 + 0.17i & 0.02 + 0.37i \\ -5.59 + 16.79i & 9.06 - 28.46i & -1.47 + 5.59i & -1.59 + 4.72i & 0.49i & -0.01 + 0.72i \\ -0.09 + 0.27i & -1.47 + 5.59i & 3.57 - 8.99i & -0.88 + 2.44i & 0.24i & -0.02 + 0.21i \\ -0.42 + 1.53i & -1.59 + 4.72i & -0.8 + 2.4i & 3.60 - 11.81i & 0.06 + 1.37i & -0.03 + 1.26i \\ 0.01 + 0.17i & -0.01 + 0.49i & -0.01 + 0.24i & 0.06 + 1.37i & 0.14 - 2.93i & -0.01 + 0.52i \\ 0.02 + 0.37i & -0.01 + 0.72i & -0.02 + 0.21i & -0.03 + 1.26i & -0.01 + 0.52i & 0.40 - 3.28i \end{pmatrix} \quad (9)$$

Table 4. Line impedance of the reduced IEEE 30-bus network only with generator nodes.

Initial Node	End Node	Line Resistance (Ω)	Line Reactance (Ω)
25	26	1.785	0.014
25	27	115.811	0.893
25	28	16.771	0.161
25	29	-19.719	1.540
25	30	-17.164	0.707
26	27	4.399	0.044
26	28	6.425	0.051
26	29	1.425	0.541
26	30	2.366	0.369
27	28	13.060	0.096
27	29	6.056	1.114
27	30	36.224	1.231
28	29	-3.093	0.193
28	30	2.067	0.211
29	30	5.242	0.507

Set 100 MW and 100 MVar as the benchmark of active and reactive power, and then the output active power of six generators before and after simplification are as shown in Figure 12 and the power flow data of preserved nodes before and after reduction are as shown in Table 5 to test the dynamic consistency of generator nodes. As shown in Figure 12, a small disturbance is added at the generator of node 25 when $t = 0$ s, and it was removed at $t = 0.05$ s. It can be seen that for the networks before and after simplification, the steady-state characteristics of the six reserved generator nodes are almost equivalent. However, before reaching the steady state, the overshoot of the simplified network would be slightly lower than that of the original network, which is mainly caused by the reduction in network nodes and system order. Nevertheless, the difference is still within the allowable range, and the simulation results are basically in line with expectations.

Table 5. The power flow data of preserved nodes before and after reduction.

Types of Model	Number of Nodes	Voltage		Generation	
		Amplitude (kV)	Phase Angle (deg)	P_G (p.u.)	Q_G (p.u.)
Before Kron reduction	25	100.00	0.000	1.3993	-0.3316
	26	100.00	-3.339	0.5756	0.2709
	27	100.00	-10.407	0.2456	0.5338
	28	100.00	-7.498	0.3500	0.6491
	29	100.00	-6.876	0.1793	0.1329
	30	100.00	-9.132	0.1691	0.2001
After Kron reduction	25	100.00	0.000	1.3990	-0.3320
	26	100.20	-3.339	0.5758	0.2712
	27	100.50	-10.407	0.2457	0.5337
	28	100.50	-7.498	0.3501	0.6488
	29	100.00	-6.786	0.1794	0.1328
	30	100.00	-9.132	0.1689	0.2002

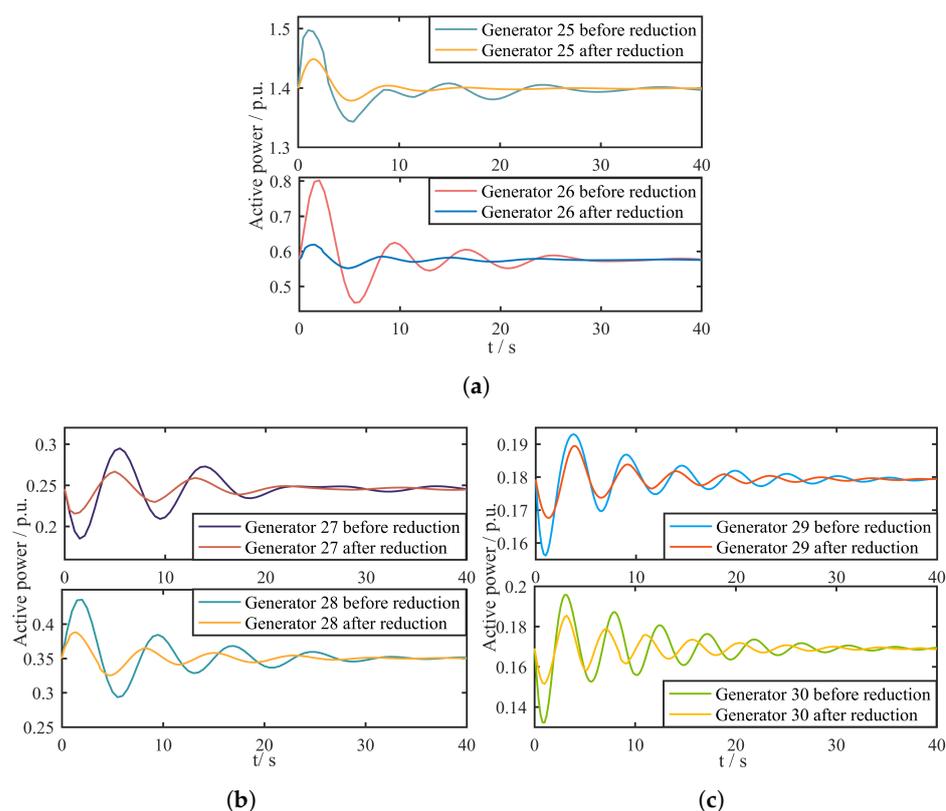


Figure 12. Output active power before and after simplification of the generator nodes 25–30.

5. Conclusions

In this paper, an improved Kron reduction method with node ordering optimization is proposed to achieve the selective elimination of flexible load buses in order to meet different requirements of power system calculation and dispatching. By classifying and ordering nodes according to users' requirements, the electrical network can be simplified "node by node". Therefore, controllable and flexible load buses, such as those in air conditioning and electric vehicles, can be preserved to make full use of their full potential in the elastic demand response. This makes Kron reduction more suitable for large-scale power network control and scheduling. To prove the effectiveness of the proposed Kron reduction method, IEEE 5-bus and 30-bus are simplified, respectively. In the IEEE 5-bus system, the reduction details are shown to illustrate that the added branches in the Kron reduction with order optimization are fewer than that without order optimization. In addition, the output active power of generator nodes and the power flow data in both the IEEE 5-bus and 30-bus systems of preserved nodes before and after reduction can fully verify the equivalency and dynamic consistency of the models. In practical applications, Kron reduction based on node ordering optimization can be used to simplify the actual complex network topology of low-voltage distribution systems, so that the non-generator adjustable nodes can be preserved after simplification, and uncontrollable nodes can be eliminated. Then, we can cluster controllable distributed units according to the heterogeneous characteristics, which aims to manage a large number and small volume of distributed units and thus make use of their potential in power system dispatching. Furthermore, the follow-up study of our paper may focus on a more intelligent node ordering method for flexible load buses.

Author Contributions: Data curation, H.S.; formal analysis, L.H.; investigation, Y.W., writing—review and editing, W.W.; data curation, Y.Q. All authors have read and agreed to the published version of the manuscript.

Funding: This work is funded by the National Natural Science Foundation of China (No. 61773137), the Natural Science Foundation of Shandong Province (Nos. ZR2019MF030 and ZR2018PEE018) and the China Postdoctoral Science Foundation (No. 2018M641830).

Data Availability Statement: Not applicable.

Acknowledgments: The authors would like to thank the anonymous reviewers for their valuable comments.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Xu, B.; Luo, Y.; Xu, R.; Chen, J. Exploring the driving forces of distributed energy resources in China: Using a semiparametric regression model. *Energy* **2021**, *236*, 121452. [\[CrossRef\]](#)
2. Hoque, M.M.; Khorasany, M.; Razzaghi, R.; Wang, H.; Jalili, M. Transactive Coordination of Electric Vehicles with Voltage Control in Distribution Networks. *IEEE Trans. Sustain. Energy* **2021**, *13*, 391–402. [\[CrossRef\]](#)
3. Zhao, D.; Wang, H.; Tao, R. Multi-time Scale Dispatch Approach for an AC/DC Hybrid Distribution System Considering the Response Uncertainty of Flexible Loads. *Electr. Power Syst. Res.* **2021**, *199*, 107394. [\[CrossRef\]](#)
4. Li, Z.; Sun, Z.; Meng, Q.; Wang, Y.; Li, Y. Reinforcement learning of room temperature set-point of thermal storage air-conditioning system with demand response. *Energy Build.* **2022**, *259*, 111903. [\[CrossRef\]](#)
5. Batt Apothula, G.; Yammani, C.; Maheswarapu, S. Multi-objective simultaneous optimal planning of electrical vehicle fast charging stations and DGs in distribution system. *J. Mod. Power Syst. Clean Energy* **2019**, *7*, 923–934. [\[CrossRef\]](#)
6. Hlalele, T.G.; Zhang, J.; Naidoo, R.M.; Bansal, R.C. Multi-objective economic dispatch with residential demand response programme under renewable obligation. *Energy* **2021**, *218*, 119473. [\[CrossRef\]](#)
7. Landeros, A.; Koziel, S.; F Abdel-Fattah, M. Distribution network reconfiguration using feasibility-preserving evolutionary optimization. *J. Mod. Power Syst. Clean Energy* **2019**, *7*, 589–598. [\[CrossRef\]](#)
8. Ashraf, S.M.; Rathore, B.; Chakrabarti, S. Performance analysis of static network reduction methods commonly used in power systems. In Proceedings of the 2014 Eighteenth National Power Systems Conference (NPSC), Guwahati, India, 18–20 December 2014.
9. Garvey, S.; Penny, J.; Gilbert, A. The reduction of computation effort in Kron’s methods for eigenvalue and response analysis of large structures. *Comput. Struct.* **1990**, *34*, 593–602. [\[CrossRef\]](#)
10. Degeneff, R.C.; Gutierrez, M.; Salon, S.; Burow, D.; Nevins, R. Kron’s reduction method applied to the time stepping finite element analysis of induction machines. *IEEE Trans. Energy Convers.* **1995**, *10*, 669–674. [\[CrossRef\]](#)
11. Dorfler, F.; Bullo, F. Kron reduction of graphs with applications to electrical networks. *IEEE Trans. Circuits Syst. I Regul. Pap.* **2012**, *60*, 150–163. [\[CrossRef\]](#)
12. Caliskan, S.Y.; Tabuada, P. Kron Reduction of Generalized Electrical Networks. *arXiv* **2012**, arXiv:1207.0563.
13. Dhople, S.V.; Johnson, B.B.; Dörfler, F.; Hamadeh, A.O. Synchronization of nonlinear circuits in dynamic electrical networks with general topologies. *IEEE Trans. Circuits Syst. I Regul. Pap.* **2014**, *61*, 2677–2690. [\[CrossRef\]](#)
14. Monshizadeh, N.; De Persis, C.; van der Schaft, A.J.; Scherpen, J.M. A novel reduced model for electrical networks with constant power loads. *IEEE Trans. Autom. Control.* **2017**, *63*, 1288–1299. [\[CrossRef\]](#)
15. Floriduz, A.; Tucci, M.; Rivero, S.; Ferrari-Trecate, G. Approximate Kron reduction methods for electrical networks with applications to plug-and-play control of AC islanded microgrids. *IEEE Trans. Control. Syst. Technol.* **2018**, *27*, 2403–2416. [\[CrossRef\]](#)
16. Abdolmaleki, B.; Shafiee, Q. Online kron reduction for economical frequency control of microgrids. *IEEE Trans. Ind. Electron.* **2019**, *67*, 8461–8471. [\[CrossRef\]](#)
17. Souza, B.; Araujo, L.; Penido, D. An Extended Kron Method for Power System Applications. *IEEE Lat. Am. Trans.* **2020**, *18*, 1470–1477. [\[CrossRef\]](#)
18. Pecenak, Z.K.; Disfani, V.R.; Reno, M.J.; Kleissl, J. Multiphase distribution feeder reduction. *IEEE Trans. Power Syst.* **2017**, *33*, 1320–1328. [\[CrossRef\]](#)
19. Zhang, X.; Cheng, H.; Grijalva, S.; Masoud, B. A Distributed Computation Method for Multi-area ATC Evaluation Based on Kron Reduction. *Autom. Electr. Power Syst.* **2013**, *37*, 68–74.
20. Ploussard, Q.; Olmos, L.; Ramos, A. An efficient network reduction method for transmission expansion planning using multicut problem and Kron reduction. *IEEE Trans. Power Syst.* **2018**, *33*, 6120–6130. [\[CrossRef\]](#)